

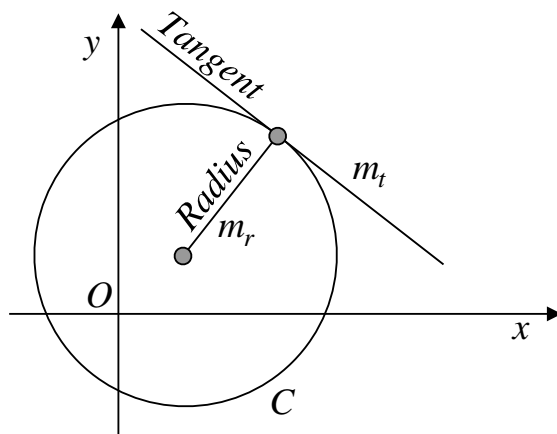
Lesson 9

A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

9.1 Tangents

A tangent to a circle is a straight line that touches the circle at a single point. If a radius of the circle is drawn to the point touched by the tangent, then that radius makes a right angle with the tangent.

In other words, the radius and the tangent are mutually perpendicular.



If the tangent has gradient m_t and the perpendicular radius has gradient m_r , then *each is the sign changed reciprocal of the other*.

That is,

$$m_t \times m_r = -1$$

Keeping this relationship between m_t and m_r in mind is often the key to answering a question about a circle that involves a tangent.

9.2 Example #1

In the above diagram, suppose that the equation of the radius is,

$$y = \frac{3}{2}x - \frac{1}{2}$$

and that the point where radius and tangent meet is $(6, 5)$.

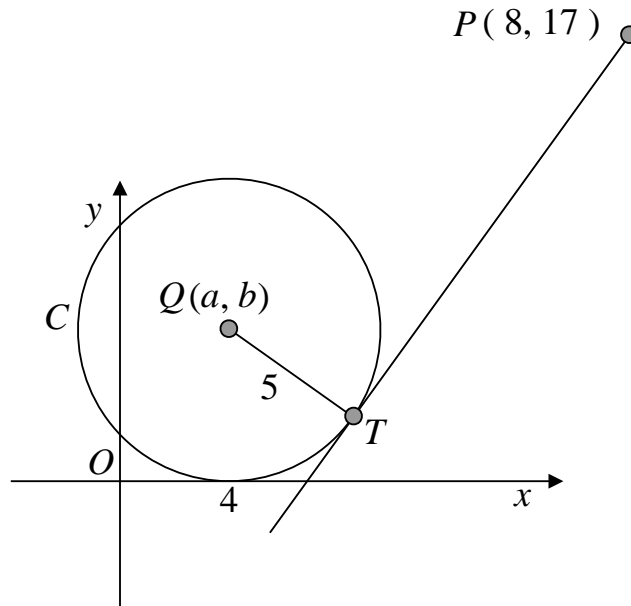
What is the equation of the tangent ?

Teaching Video : <http://www.NumberWonder.co.uk/v9033/9a.mp4>



[2 marks]

9.3 Example #2



A circle C , with centre $Q(a, b)$ and radius 5, touches the x -axis at $(4, 0)$.

(i) Write down the value of a and the value of b .

(ii) Find a Cartesian equation of C .

A tangent to the circle, drawn from the point $P(8, 17)$, touches the circle at T .

(iii) Find, to 3 significant figures, the length of PT .

Teaching Video : <http://www.NumberWonder.co.uk/v9033/9b.mp4>



[7 marks]

9.4 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 86

Question 1

A circle has equation,

$$(x - 1)^2 + (y + 2)^2 = 13$$

- (i) State the coordinates of the centre of the circle.

[1 mark]

- (ii) Show that the point (3, 1) is on this circle.

[2 marks]

- (iii) What is the gradient of the radius of the circle to the point (3, 1) ?

[2 marks]

A tangent to the circle touches the point (3, 1)

- (iv) Find the equation of this tangent in the form $y = mx + c$

[3 marks]

Question 2

A circle has equation,

$$(x + 5)^2 + (y - 1)^2 = 65$$

The point (3, 2) is on this circle

Find the equation of the tangent to the circle at the point (3, 2)

[5 marks]

Question 3

A circle has equation,

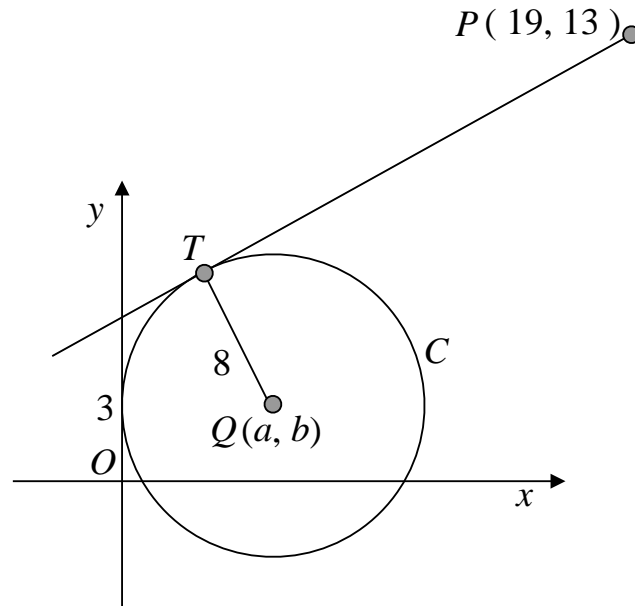
$$x^2 + y^2 - 6x + 4y = 7$$

The point (1, 2) is on this circle.

Find the equation of the tangent to the circle at the point (1, 2)

[6 marks]

Question 4



A circle C , with centre $Q(a, b)$ and radius 8, touches the y -axis at $(0, 3)$

(i) Write down the value of a and the value of b

[2 marks]

(ii) Find a Cartesian equation of C

[2 marks]

A tangent to the circle, drawn from the point $P(19, 13)$, touches the circle at T

(iii) Find, to 3 significant figures, the length of PT

[3 marks]

Question 5

A circle has equation,

$$(x - 2)^2 + (y + 1)^2 = 16$$

- (i) Show that the point (8, 5) is NOT on the circle.

[2 marks]

- (ii) Find the length of a tangent from the point (8, 5) to the circle.

[5 marks]

Question 6

A-Level Examination Question from January 2013, Paper C2, Q5 (Edexcel)

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M

- (a) Find
- (i) the coordinates of the point M

- (ii) the radius of the circle C

[5 marks]

N is the point with coordinates $(25, 32)$

- (b) Find the length of the line MN

[2 marks]

The tangent to C at a point P on the circle passes through point N

- (c) Find the length of the line NP

[2 marks]

Question 7

A-Level Examination Question from January 2018, Paper C12, Q11 (Edexcel)

The circle C has equation

$$x^2 + y^2 - 8x - 10y + 16 = 0$$

The centre of C is at the point T

- (a) Find,
(i) the coordinates of the point T

[2 marks]

- (ii) the radius of the circle C

[1 mark]

The point M has coordinates (20, 12)

- (b) Find the exact length of the line MT

[2 marks]

Point P lies on the circle C such that the tangent at P passes through the point M

- (c) Find the exact area of triangle MTP , giving your answer as a simplified surd.

[3 marks]

Question 8

Additional Mathematics Examination Question from June 2016, Q12 (OCR)

The line L_1 has equation $3x - y = 1$ and the point P has coordinates $(8, 3)$

- (i) Find the equation of the line L_2 which passes through P and is perpendicular to line L_1

[3 marks]

- (ii) Find the coordinates of the point Q where L_1 and L_2 intersect

[3 marks]

- (iii) Find length PQ

[2 marks]

- (iv) Write down the equation of the circle that has centre P and line L_1 as a tangent

[1 mark]

- (v) Find the equation of the other line that is a tangent to the circle and is parallel to line L_1

[3 marks]

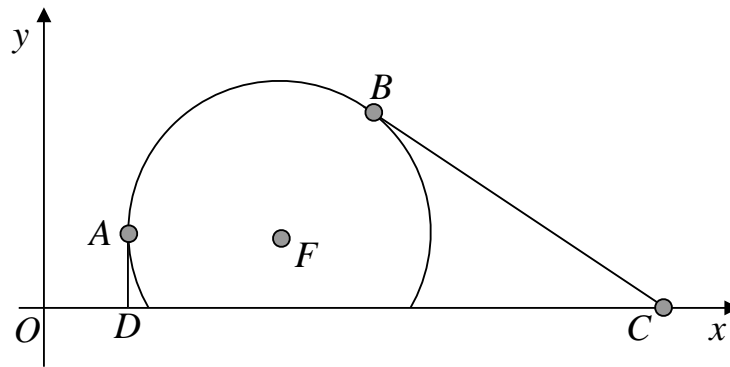
Question 9

Additional Mathematics Examination Question from June 2004, Q12 (OCR)

The shape shown in the diagram is part of a circle. The centre of the circle is $F(8, 4)$ and AD and BC are tangents at A and B respectively.

A is the point $(3, 4)$ and B is the point $(11, 8)$

A wire is stretched from D to A , round the circumference to the circle to B and then to C , where D and C are on the x -axis. Units are centimetres.



(a) Find the equation of the circle.

(b) (i) Find the gradient of FB and hence the equation of the tangent BC .

[3 marks]

(ii) Given that the length of the wire from A to B in contact with the circle is 11.07 cm, correct to 2 decimal places, find the total length of the wire.

[4 marks]

[5 marks]

Question 10

Additional Mathematics Examination Question from, June 2018, Q11 (OCR)

A circle has centre $(0, 3)$ and radius 3

- (i) Show that the equation of the circle is $x^2 + y^2 - ky = 0$ where k is to be determined.

[2 marks]

The line $y = mx - 2$ passes through the point $P(0, -2)$ and is a tangent to the circle

- (ii) Find the two possible values of m

[6 marks]

The two tangents from P meet the circle at the points A and B respectively.

- (iii) Find the lengths PA and PB

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk