Lesson 3

GCSE Mathematics
Algebraic Fractions

3.1 Algebra in the Denominator

As previously observed, to add or subtract two fractions, they must first be numerically adjusted to have a common denominator. When the denominator is algebraic, rather than numeric, the adjustment required is also algebraic.

3.2 Example

Simplify the following expression;

$$\frac{3}{(x+1)} + \frac{5}{(x+2)}$$

[4 marks]

3.3 Try

Try to simplify the following expression.
Once done, take a look at the answer, over the page.

$$\frac{7}{(x+4)} + \frac{3}{(x+3)}$$

[4 marks]

3.4 Try Answer

$$\frac{7}{(x+4)} + \frac{3}{(x+3)}$$

$$= \frac{(x+3)}{(x+3)} \times \frac{7}{(x+4)} + \frac{3}{(x+3)} \times \frac{(x+4)}{(x+4)}$$

$$= \frac{7(x+3) + 3(x+4)}{(x+3)(x+4)}$$

$$= \frac{7x + 21 + 3x + 12}{(x+3)(x+4)}$$

$$= \frac{10x + 33}{(x+3)(x+4)}$$

Note : A mathematician would not expand the brackets in the concluding denominator

[4 marks]

3.5 Exercise

Marks Available: 64

Question 1

Simplify the following expression;

$$\frac{4}{(x+5)} + \frac{3}{(x+7)}$$

[4 marks]

Question 2

Simplify the following expression;

$$\frac{5}{(2x + 3)} + \frac{3}{(x + 5)}$$

[4 marks]

Question 3

Show that;

$$\frac{5}{(x + 4)} - \frac{3}{(x + 8)} = \frac{2(x + 14)}{(x + 4)(x + 8)}$$

[4 marks]

Question 4

Show that,

$$\frac{8}{(x+4)} + \frac{3}{(x+1)} = \frac{11x+20}{(x+4)(x+1)}$$

[4 marks]

Question 5

Simplify fully the following expression;

$$\frac{x}{x+2} + \frac{1}{x}$$

[4 marks]

Question 6

GCSE Examination Question from January 2007, 3H, Q16 (Edexcel)

Simplify

(a) $\dfrac{x^2 - 3x}{2x - 6}$

[3 marks]

(b) $\dfrac{2}{x - 1} - \dfrac{3}{x}$

[3 marks]

Question 7

Show that

$$\frac{4}{x} + \frac{x}{x + 1} = \frac{(x + 2)^2}{x(x + 1)}$$

[4 marks]

Question 8

GCSE Examination Question from January 2012, 3H, Q20 (Edexcel)

Simplify fully

$$\frac{4}{x} + \frac{3}{2 - x}$$

[3 marks]

Question 9

GCSE Examination Question from November 2009, 3H, Q19 (Edexcel)

(a) Simplify, $\frac{x^2}{x^2 - 2x}$

[2 marks]

(b) Simplify, $\frac{2}{2x - 1} - \frac{1}{x + 1}$

[4 marks]

Question 10

Expand the brackets;

$$(x - 7)(x + 5)$$

[2 marks]

Question 11

Solve these equations without using a calculator;

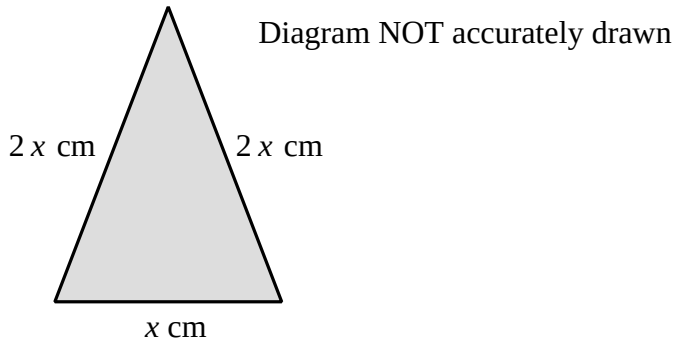
(i) $8x + 1 = 5$ **(ii)** $3 + 5x = 17$

[2, 2 marks]

Question 12

GCSE Examination Question from November 2007, 3H, Q3 (Edexcel)

A triangle has two equal sides of length $2x$ cm and one side of length x cm.



The perimeter of this triangle is 12 cm.

(i) Use this information to write down an equation in x.

(ii) Solve your equation to find the value of x.

[3 marks]

Question 13

Expand the brackets

$$(5x + 8)(3x + 7)$$

[2 marks]

Question 14

A rectangle has a length, in cm, of $x + 7$ and a height of 6 cm.

It has a perimeter is 32 cm.

(i) Calculate the value of x.

(ii) Draw the rectangle full size.

(iii) What is the area of the rectangle ?

[4 marks]

Question 15

Expand the brackets

$$(2x + 9)(5x - 6)$$

[2 marks]

Question 16

GCSE Examination Question from June 2010, 4H, Q7 (Edexcel)

Rectangular tiles have width $(x + 1)$ cm and height $(5x - 2)$ cm.

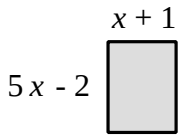


Diagram NOT accurately drawn

Some of these tiles are used to form a large rectangle.
The large rectangle is 7 tiles wide and 3 tiles high.

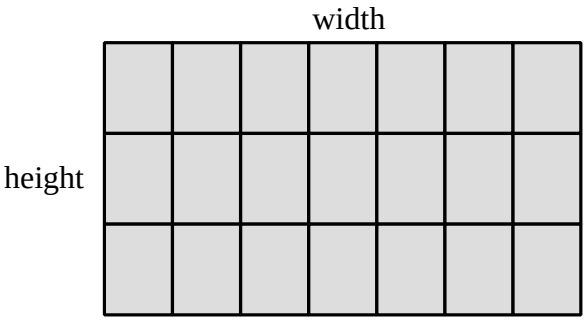


Diagram NOT accurately drawn

The perimeter of the large rectangle is 68 cm.

(a) Write down an equation in x.

[3 marks]

(b) Solve this equation to find the value of x.

[3 marks]

Question 17

Expand the brackets

$$(3x + 5)^2$$

[2 marks]