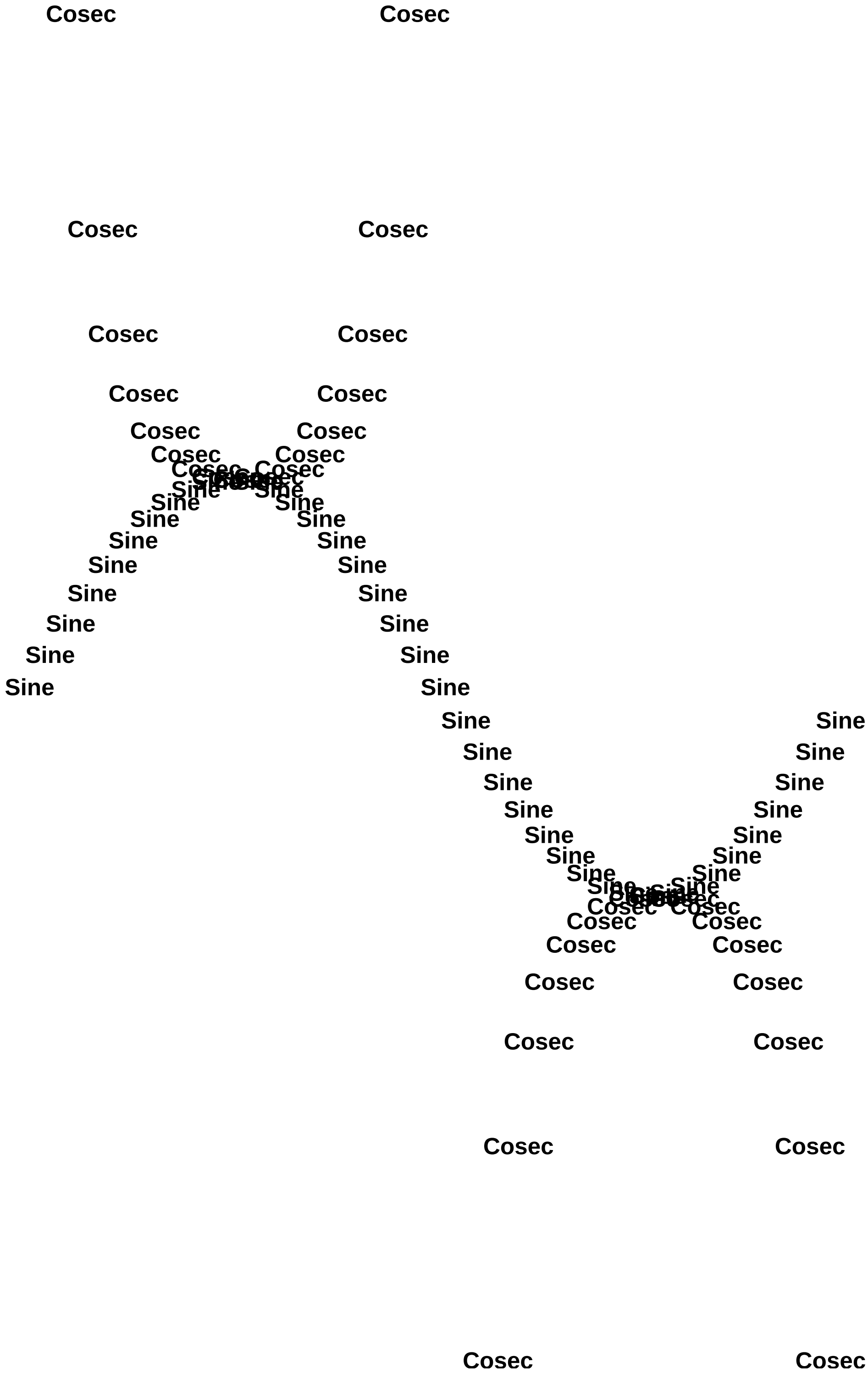
A-Level Pure Mathematics
Year 2

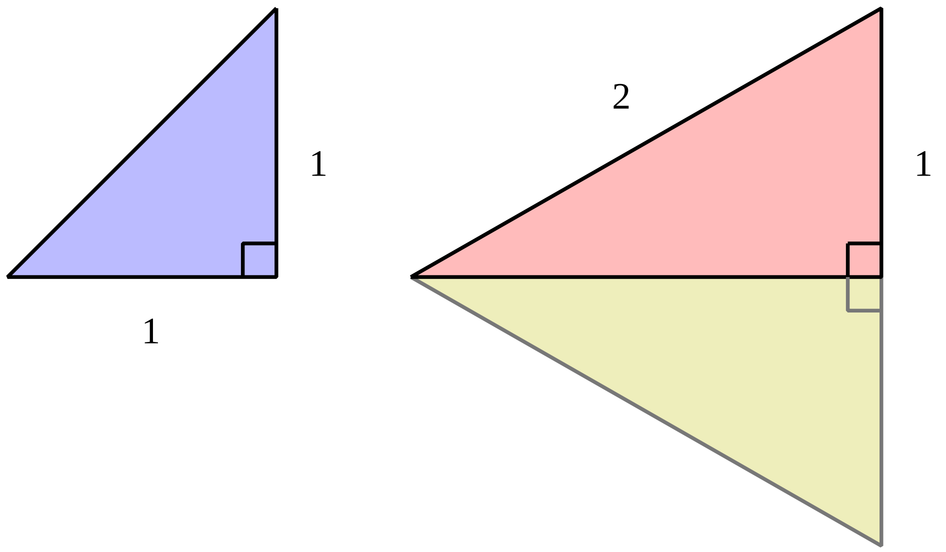
T R I G O N O M E T R Y   V

Trigonometric Identities



1.1 Trigonometric "Exact Values"

The following two simple right angled triangles are of great utility, for they can be used to determine exact values for the *sin*, *cos* and *tan* of certain angles.



The left triangle has a base of length exactly 1 unit and a height of exactly 1 unit. On this triangle in the appropriate place write the exact length of the hypotenuse and the exact size of the two non right angled angles.

The right triangle is equilateral with sides of length exactly 2 units, but has been cut in half. The focus is upon the upper half which has a hypotenuse of exact length 2 units and height of exactly 1 unit. On this triangle in the appropriate place write the exact length of the base and the exact size of the two non right angled angles.

Complete the following "Exact Values" table without using a calculator,

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

[5 marks]

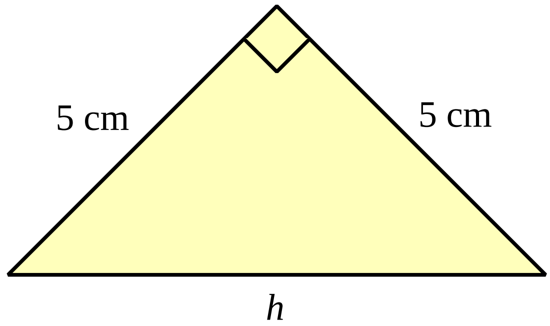
1.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

A right-angled isosceles triangle has two sides of length 5 cm.

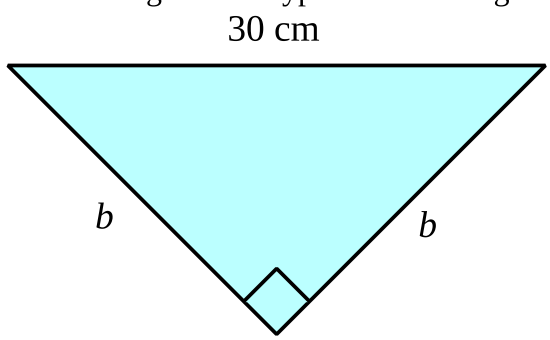


Use the theorem of Pythagoras to find the exact length of its hypotenuse, *h*, in as simplified a form as possible.

[2 marks]

Question 2

A right angled isosceles triangle has a hypotenuse of length 30 cm.

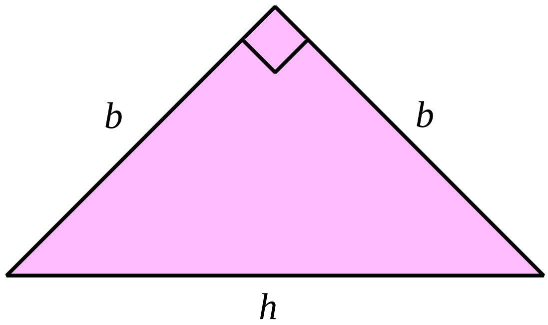


Use the theorem of Pythagoras to find the exact length of one of the other sides, *b*, in as simplified a form as possible.

[3 marks]

Question 3

A right-angled isosceles triangle has two sides of length b cm.



(i) Use the theorem of Pythagoras to find the exact length of its hypotenuse, h, in terms of b, in as simplified a form as possible.

[2 marks]

(ii) Rearrange your part (i) answer to make b the subject of the formula and present your answer with a rational denominator.

[2 marks]

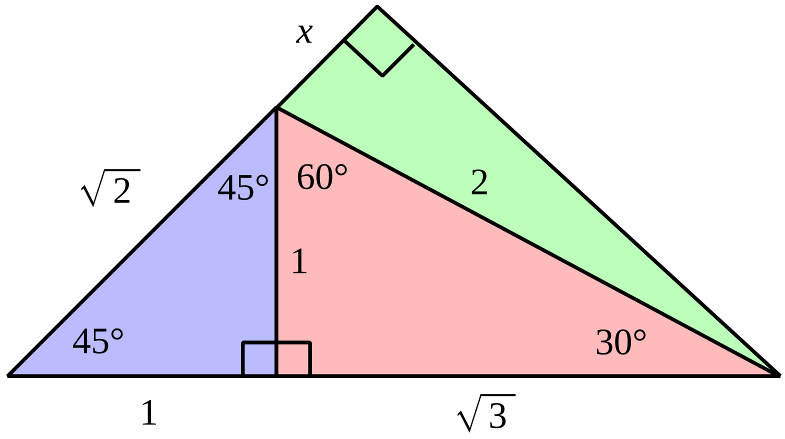
(iii) Given that a right-angled isosceles triangle has two sides of length $1 + \sqrt{5}$, write down the exact length of the hypotenuse by using part (i)'s answer.

[1 mark]

(iv) Given that a right-angled isosceles triangle has a hypotenuse of $1 + \sqrt{3}$, write down the exact length of each of the other sides using part (ii)'s answer. Any denominator in your final answer should be rational.

[2 marks]

Question 4



In the diagram the previous two triangles have been placed back to back.
An extra green triangle has been added such that the entire shape is a right-angled isosceles triangle with two angles of 45°

(i) On the diagram write in the values of the two non right-angled angles of the green triangle.

[1 mark]

(ii) By considering the entire shape, which has a hypotenuse of exact length $1 + \sqrt{3}$, write down an expression for the length of either of the other sides.

[1 mark]

(iii) Hence, or otherwise, determine the exact length of the side of the green triangle marked *x*

[3 marks]

(iv) Without using a calculator, find an exact value for *sin* 15°

[2 marks]

(v) Without using a calculator, complete the following "Exact Values" table. Show necessary working, especially for *tan* θ.

	$15°$	$75°$
$\sin\theta$		
$\cos\theta$		
$\tan\theta$		

[5 marks]

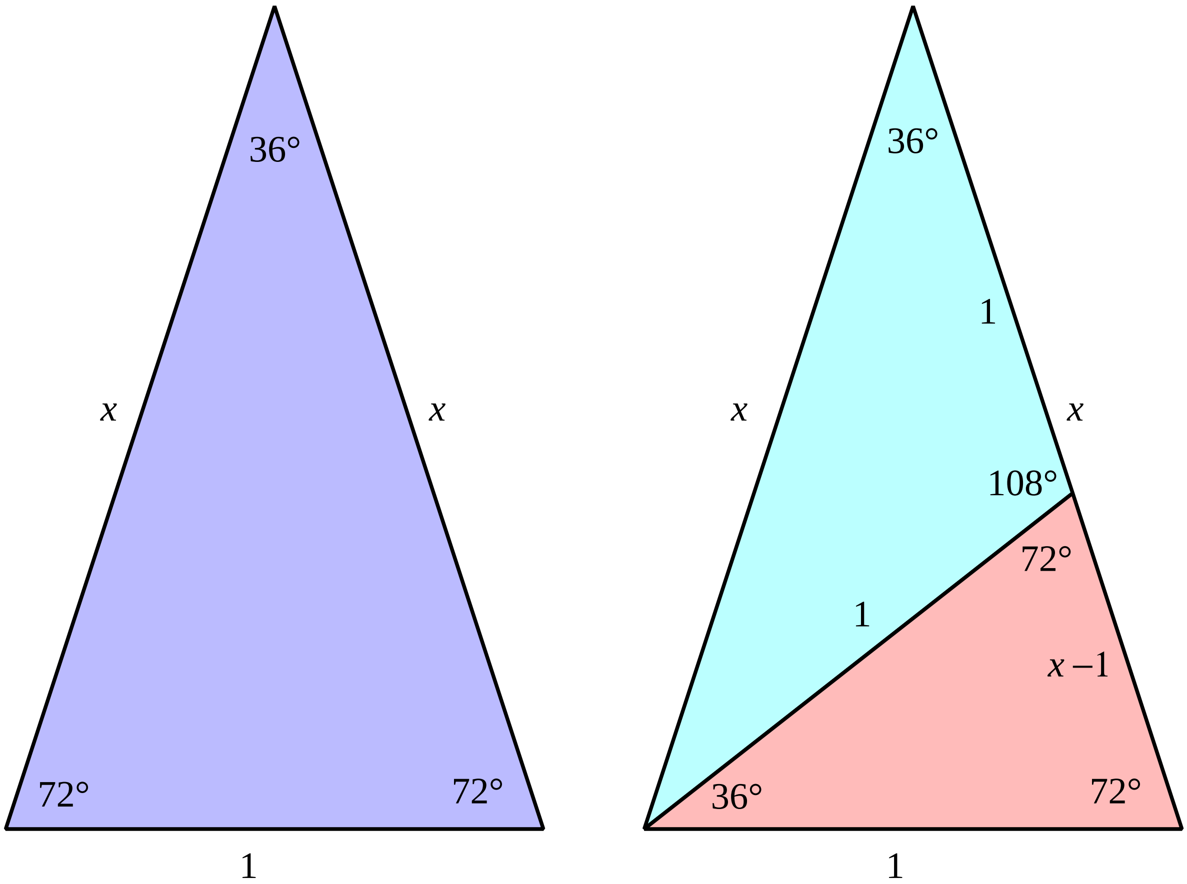
(vi) The cosine rule (reversed) states that in any triangle *ABC*,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Apply this to a suitable triangle to obtain an exact expression for *cos* 105°

[3 marks]

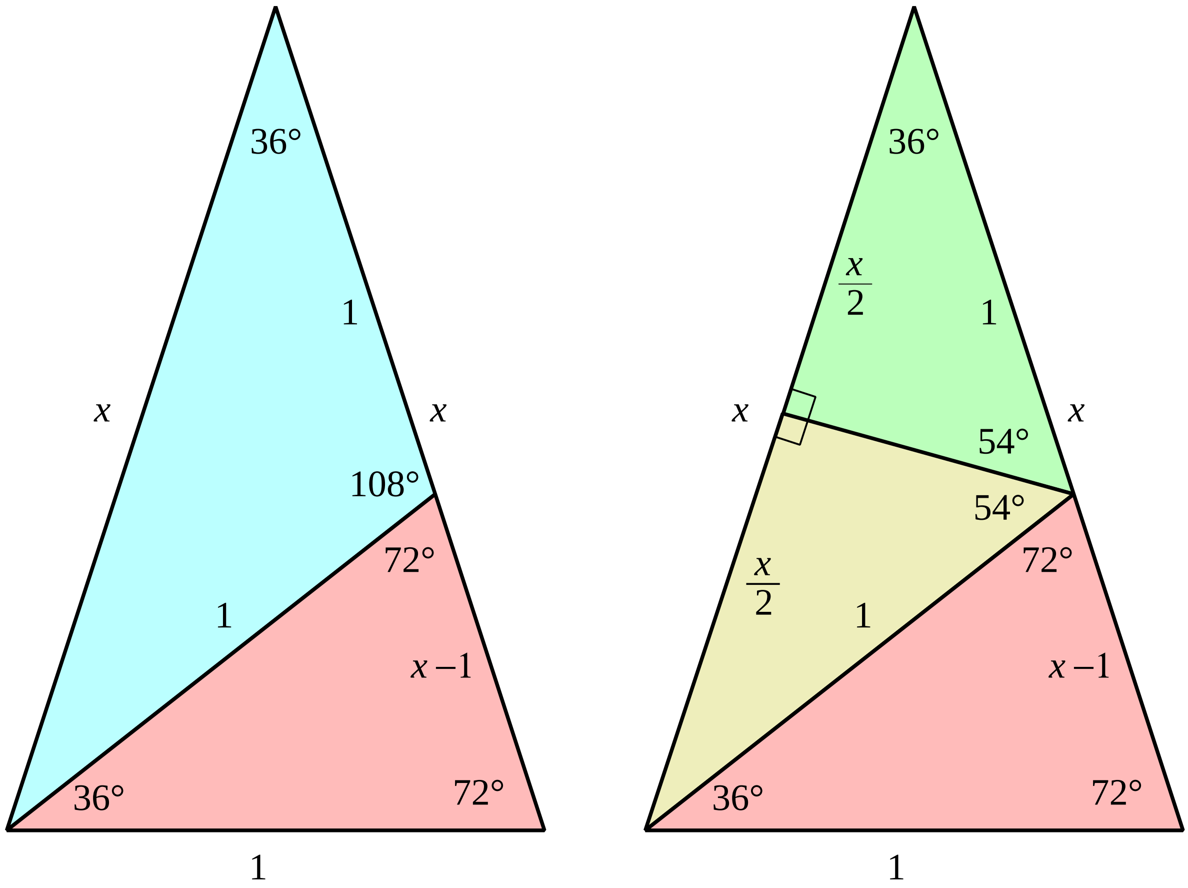
Question 5



To the left is drawn an isosceles purple triangle with base angles of 72°.
The base is 1 unit in length and the congruent sides are of length *x*.
The angle bisector of the bottom-left base angle is drawn, cutting the original triangle into two triangles, one pink, one blue, as shown right.
These remarkable property of both being isosceles, and so the pink triangle has two sides that must be of length 1 which in turn forces the blue triangle to also have two sides of length 1.
This then means that the base of the pink triangle is $x - 1$

(i) The original blue triangle and the new pink triangle are similar because they both have angles of 72°, 72° and 36°.
Use this fact to find the exact numerical value of x

[5 marks]



The bisector of the 108° angle in the blue triangle is now drawn.
This results in two congruent right angled triangles, one green and one yellow.

(ii) Use the diagram above right, and your knowledge of right angled trigonometry, to find an exact value for *cos* 36°

[3 marks]

(iii) Show that the exact value of $\sin 36$ is $\dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$

[3 marks]

Question 6
Tom is wondering about how to expand the brackets of,

$$\sin\ (A + B)$$

Lucy suggests that it might always be,

$$\sin A + \sin B$$

for all values of *A* and *B*.

(i) Show that Lucy's suggestion is not correct by finding suitable "counterexample" values for *A* and *B* taken from an "Exact Values" table.

[2 marks]

Prof Triggy Brain tells Tom that the correct expansion is;

$$\sin\ (A + B) = \sin A \cos B + \cos A \sin B$$

(ii) Provide an illustration that this could be correct by picking suitable values for *A* and *B* from an "Exact Values" table.

[2 marks]

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