

Lesson 3

A-Level Pure Mathematics : Year 2 Trigonometric Identities

3.1 The Double Angle Formulae

In the A-Level examination, candidates are given The Addition Formulae but not The Double Angle Formulae. They should be memorised !

The Double Angle Formulae

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

3.2 Proof : For $\sin 2A$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\text{Let } B = A$$

$$\sin(A + A) = \sin A \cos A + \cos A \sin A$$

$$\sin 2A = 2 \sin A \cos A \quad \square$$

3.3 Proof : For $\cos 2A$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\text{Let } B = A$$

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A \quad \square$$

3.4 Proof : For $\tan 2A$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{Let } B = A$$

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad \square$$

3.5 Example

Prove that,
$$\frac{\cos 2A}{1 + \sin 2A} = \frac{1 - \sin 2A}{\cos 2A}$$

Teaching Video : <http://www.NumberWonder.co.uk/v9040/3.mp4>



Watch the Teaching Video, write out the proof.



[5 marks]

3.6 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*
Marks Available : 30

Question 1

By changing the 1 for $\cos^2 A + \sin^2 A$, or otherwise, prove this identity;

$$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$

LHS =

[3 marks]

Question 2

By changing both 1s for $\cos^2 A + \sin^2 A$, or otherwise, prove this identity;

$$\sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}} = \tan A$$

LHS =

[3 marks]

Question 3

By use of a difference of two squares, or otherwise, prove this identity;

$$\frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$$

LHS =

[3 marks]

Question 4

By use of a difference of two squares, or otherwise, prove this identity;

$$\cos^4 A - \sin^4 A = \cos 2A$$

LHS =

[3 marks]

Question 5

Prove that;

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

If you need help with this question there are several video's available as it's a "classic".
Search the internet using "cos 3A".

[6 marks]

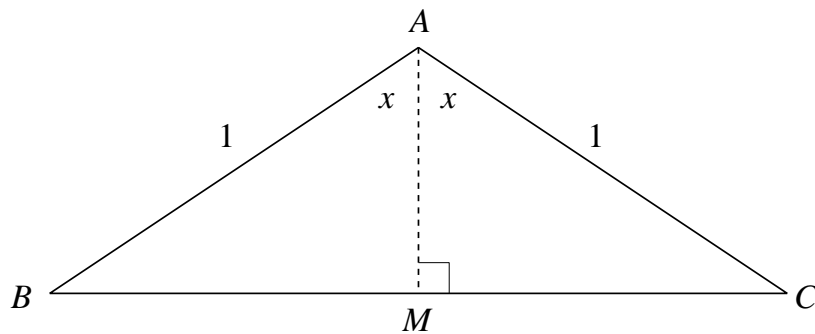
Question 6

(a) By changing the 1 for $\cos^2 A + \sin^2 A$, prove this identity;

$$1 - 2 \sin^2 x = \cos 2x$$

[2 marks]

(b)



ABC is an isosceles triangle

$AB = AC = 1$

M is the midpoint of BC

(i) Use trigonometry to find an expression, in terms of x , for BM

[1 mark]

(ii) Hence write down an expression, in terms of x , for BC

[1 mark]

(iii) Use the cosine rule to find an expression, in terms of $\cos 2x$, for BC^2

[2 marks]

(iv) Hence show that $\cos 2x = 1 - 2 \sin^2 x$

[2 marks]

Question 7

Prove that;

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

[4 marks]