Lesson 6

A-Level Pure Mathematics : Year 2
Trigonometric Identities

6.1 Examination Questions

A typical examination question will revolve around the solving of a trigonometric
equation that is in essence a quadratic equation. In order to obtain the quadratic
it may well be required to use one of the equations,

$$cos^2\,\theta + sin^2\,\theta = 1$$

$$1 + tan^2\theta = sec^2\,\theta$$

$$cot^2\,\theta + 1 = csc^2\,\theta$$

6.2 Example

C3 Examination question from January 2012, Q5.

Solve, for $0 \leqslant \theta \leqslant 180°$, $\qquad 2\,cot^2\,3\theta = 7\,cosec\,3\theta - 5$

Give your answers in degrees to 1 decimal place.

Teaching Video : http://www.NumberWonder.co.uk/v9040/6a.mp4

Watch the video from "Exam Solutions"
Write out a solution to the question.

☞

[10 marks]

6.3 Exercise

Each question comes with Video Support from "Exam Solutions"
You are not expected to watch these videos.
They are there for when you need a helping hand.

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 55

Question 1

C3 Examination question from January 2011, Q3

Find all the solutions of $2 \cos 2\theta = 1 - 2 \sin \theta$ in the interval $0 \leqslant \theta \leqslant 360°$

[6 marks]
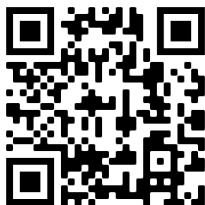
Need help with Question 1 ?

Question 2

C3 Examination question from January 2010, Q8

Solve, $cosec^2 2x - cot 2x = 1$ for $0 \leqslant x \leqslant 180°$

[7 marks]

Need help with Question 2 ?

Question 3

C3 Examination question from June 2011, Q6

(a) Prove that, $\dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} = \tan\theta,$ $\quad \theta \neq 90n°,\ n \in \mathbb{Z}$

[4 marks]

(b) Hence, or otherwise

(i) show that $\tan 15° = 2 - \sqrt{3}$

[3 marks]

(ii) solve, for $0 < x < 360°,\quad cosec\ 4\theta - \cot 4\theta = 1$

[5 marks]

Need help with Question 3 ?

http://www.NumberWonder.co.uk/v9040/6c.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9040/6d.mp4 (Part 2)
http://www.NumberWonder.co.uk/v9040/6e.mp4 (Part 3)

Part 1   Part 2   Part 3

Question 4

C3 Examination question from June 2006, Q6

(**a**)   Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$

[2 marks]

(**b**)   Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta$$

[2 marks]

(**c**)   Solve, for $90° \leqslant \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta$$

[6 marks]

Need help with Question 4 ?

Part 1   Part 2

Question 5

C3 Examination question from June 2008, Q5

(a)   Given that $sin^2 \theta + cos^2 \theta \equiv 1$, show that $1 + cot^2 \theta \equiv cosec^2 \theta$

[2 marks]

(b)   Solve, for $0 \leqslant \theta < 180°$, the equation

$$2 \, cot^2 \theta \ - \ 9 \, cosec \, \theta \ = \ 3$$

giving your answers to 1 decimal place.

[6 marks]

Need help with Question 5 ?

Part 1   Part 2

Question 6

C3 Examination question from June 2010, Q1

(a)   Show that

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

[2 marks]

(b)   Hence find, for $-180° \leqslant \theta \leqslant 180°$, all the solutions of

$$\frac{2 \sin 2\theta}{1 + \cos 2\theta} = 1$$

Give your answers to 1 decimal place.

[3 marks]

Need help with Question 6 ?
http://www.NumberWonder.co.uk/v9040/6j.mp4   (Part 1)
http://www.NumberWonder.co.uk/v9040/6k.mp4   (Part 2)

Part 1   Part 2

Question 7

C3 Examination question from January 2007, Q1

(a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

[5 marks]

(b) Given that $\sin \theta = \dfrac{\sqrt{3}}{4}$ find the exact value of $\sin 3\theta$

[2 marks]

Need help with Question 7 ?

http://www.NumberWonder.co.uk/v9040/6m.mp4   (Part 1)
http://www.NumberWonder.co.uk/v9040/6n.mp4   (Part 2)

Part 1   Part 2