6.4 Homework

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 32

Question 1

A-Level Paper 1 Examination question from June 2018, Q8

The depth of water, D metres, in a harbour on a particular day is modelled by

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6.30 am and it takes 2 hours to load its cargo. The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

[1 mark]

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

[4 marks]

Question 2

A-Level Paper 2 Examination question from June 2018, Q12 (a) edited

Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta \qquad \theta \neq \frac{(2n + 1)\ 180^\circ}{2},\ n \in \mathbb{Z}$$

[3 marks]

Question 3

A-Level C3 Examination question from June 2017, Q9

(a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x \qquad x \neq (2n + 1)90°,\ n \in \mathbb{Z}$$

[4 marks]

(b) Given that $x \neq 90°$ and $x \neq 270°$ solve for $0 \leqslant x < 360°$

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate

[5 marks]

Question 4

A-Level Paper 1 Examination question from June 2019, Q6

(a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[6 marks]

(b) Deduce the smallest positive solution to the equation

$$5 \sin(2x - 50°) = 9 \tan(x - 25°)$$

[2 marks]

Question 5

A-Level Paper 2 Examination question from June 2019, Q12

(a) Prove, $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2 \cot 2\theta \qquad \theta \neq (90n)^\circ, \ n \in \mathbb{Z}$

[4 marks]

(b) Hence solve, for $90^\circ < \theta < 180^\circ$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place

[3 marks]