Lesson 2

A-Level Pure Mathematics, Year 2
Functions II

2.1 The Modulus Function (Part 1)

$|x|$ means "*magnitude of x*" although mathematicians also say "*modulus of x*".

When the modulus function applies to a positive quantity is leaves it alone.
When applied to a negative quantity it makes it positive.

For example, $|45| = 45$

$|-62| = 62$

2.2 Modulus Sketching Rule 1

To sketch $y = |f(x)|$ **Bounce negative x**

◇ Sketch $y = f(x)$ using a dashed line for points below the x-axis.

◇ Reflect any part of the curve below the x-axis in the x-axis.

2.3 Example

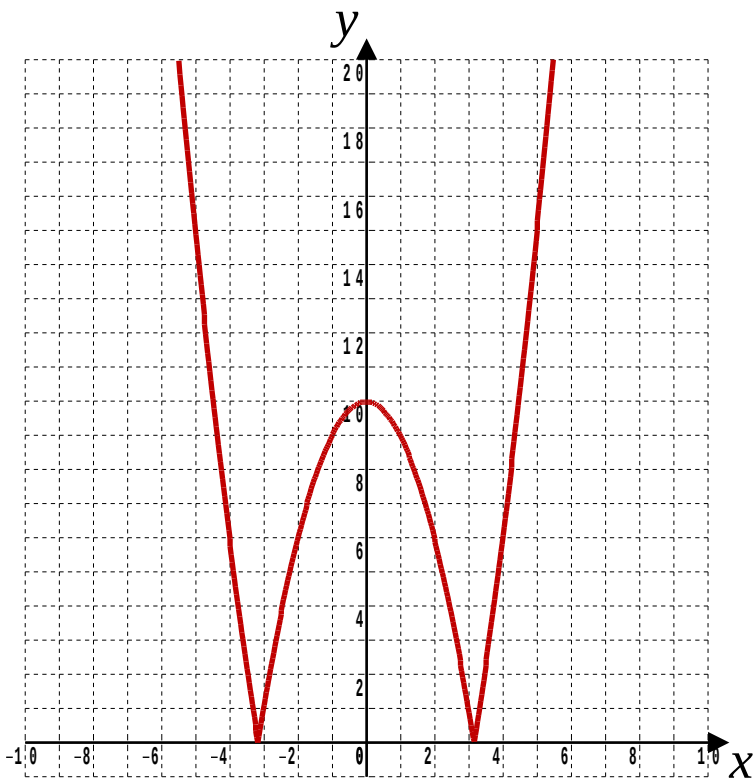
Sketch the curve,

(i) $y = x^2 - 4$ **(ii)** $y = |x^2 - 4|$

[2, 2 marks]

2.4 Solving Modulus Equations: An Example

The graph is of the function, $f(x) = |x^2 - 10|$, $x \in \mathbb{R}$



(i) From the graph, write down all the solutions to the equation $f(x) = 6$

[2 marks]

(ii) Obtain the part (i) solutions using algebra.

[4 marks]

(iii) To the graph add a plot of the function $g(x) = x + 10$

[1 mark]

(iv) Estimate, graphically, the solutions to the equation;

$$f(x) = g(x)$$

That is, $\left| x^2 - 10 \right| = x + 10$

[2 marks]

(v) Solve the equation $f(x) = g(x)$ using algebra.

[4 marks]

2.5 Watch Out !

Extraneous Solutions

When solving modulus equations it is possible to generate extra solutions and so a check that each solution satisfies the original equation is always advised.

2.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 40

Question 1

Solve these equations,

(i) $|x - 4| = 3, \quad x \in \mathbb{R}$

[2 marks]

(ii) $|x - 7| = 15, \quad x \in \mathbb{R}$

[2 marks]

(iii) $|9 - x| = 2, \quad x \in \mathbb{R}$

[2 marks]

Question 2

(i) Sketch the graph of $y = |\sin x| \quad x \in \mathbb{R}, \quad 0° \leqslant x \leqslant 360°$

[2 marks]

(ii) Hence, or otherwise, solve the equation;

$$|\sin x| = 0.5, \quad x \in \mathbb{R}, \quad 0° \leqslant x \leqslant 360°$$

[2 marks]

Question 3

A function f is defined by;

$$f : x \to |x - 2| - 3, \qquad x \in \mathbb{R}$$

Solve the equation, $f(x) = 1$

[3 marks]

Question 4

Find the exact values of x for which ;

$$\left|\frac{2}{x - 3}\right| = 3$$

[3 marks]

Question 5

Solve the equation;

$$2 - |x + 1| = \frac{1}{2}x$$

[3 marks]

Question 6

Find the exact solutions of the equation;

$$4 - x^2 = |2x - 1|$$

[5 marks]

Question 7

Solve the equation;

$$\left|\left|x-4\right|-2\right|=1$$

[5 marks]

Question 8

(i) Explain why the graph of a function of the form

$$y = f(|x|)$$

has mirror symmetry in the y-axis.

[2 marks]

(ii) Sketch the graph of;

$$y = sin(|x|) \qquad x \in \mathbb{R}, \qquad -360° \leqslant x \leqslant 360°$$

[2 marks]

(iii) Hence, or otherwise, write down the solutions to the equation;

$$sin(|x|) = \frac{\sqrt{3}}{2} \qquad x \in \mathbb{R}, \qquad -360° \leqslant x \leqslant 360°$$

[2 marks]

Question 9

Solve the equation;

$$\left| x^2 - \pi \right| = x + \pi$$

[5 marks]