4.1 The Modulus Function (Part 2)

The fact that the modulus function can take any part of an equation's graph that is below the *x*-axis, and reflects it in the *x*-axis was studied previously. Here is the rule that was used;

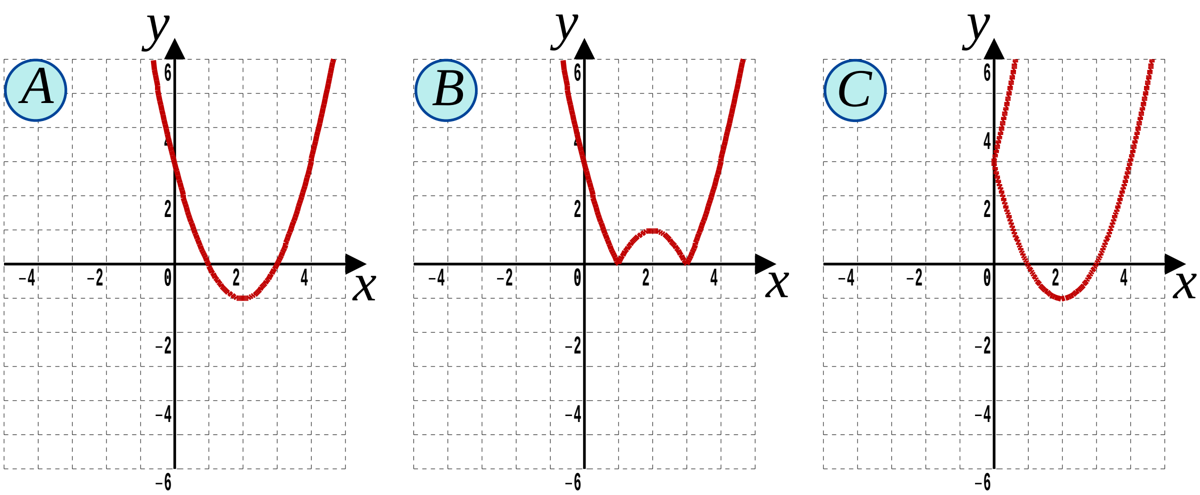
Modulus Sketching Rule 1

To sketch $y = |f(x)|$ **Bounce negative *x***

◇ Sketch $y = f(x)$ using a dashed line for points below the *x*-axis.

◇ Reflect any part of the curve below the *x*-axis in the *x*-axis.

For example, graph *A* shows the parabola with quadratic equation $y = x^2 - 4x + 3$ and graph *B* shows the effect of modulising graph *A*: That is, $y = |x^2 - 4x + 3|$



4.1.1 Example

Find the equation that will yield graph *C*, where the "bounce" is against the *y*-axis.

[6 marks]

4.2 By Hand

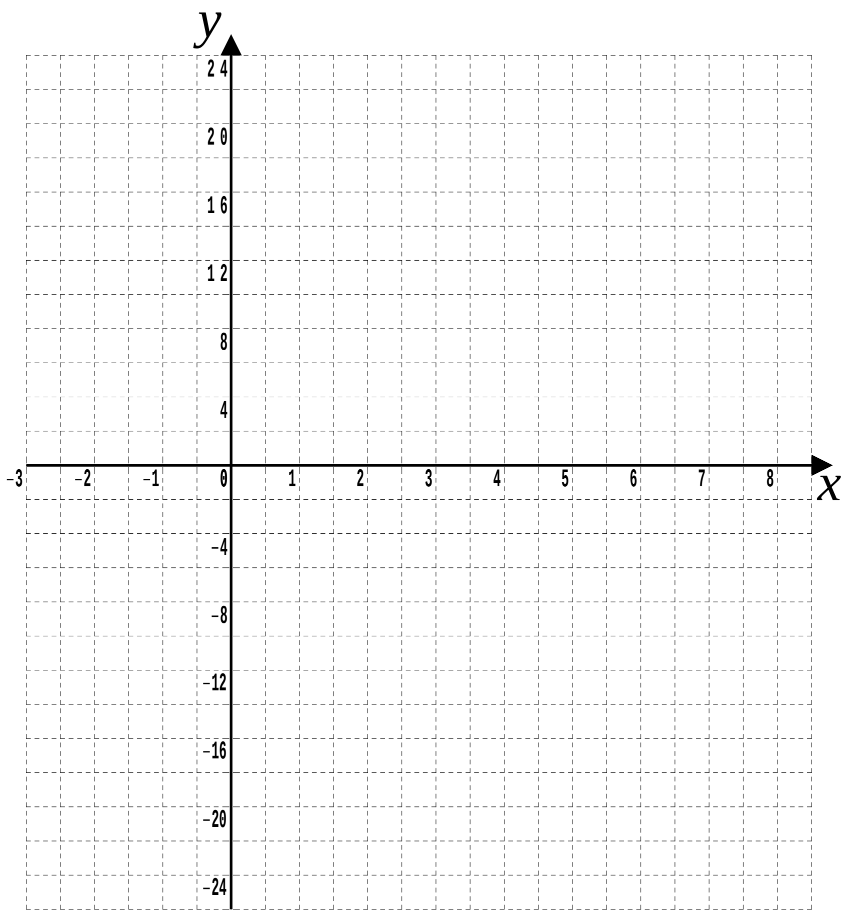
Faced with an unfamiliar modulus question it may be best to go back to basics by drawing up a table and plotting points to get a feel of how it is behaving. With such plotting, keeping one's wits sharp is essential !

4.2.1 Example

Use the table below to assist in plotting the following equation;

$$|y + 1| = |(x - 2)^2|$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
y											



[6 marks]

4.3 A Subtle Variation

Typically, an examination question will ask for two separate graphs of

◇ $y = | f(x) |$

and ◇ $y = f(| x |)$

Modulus Sketching Rule 2

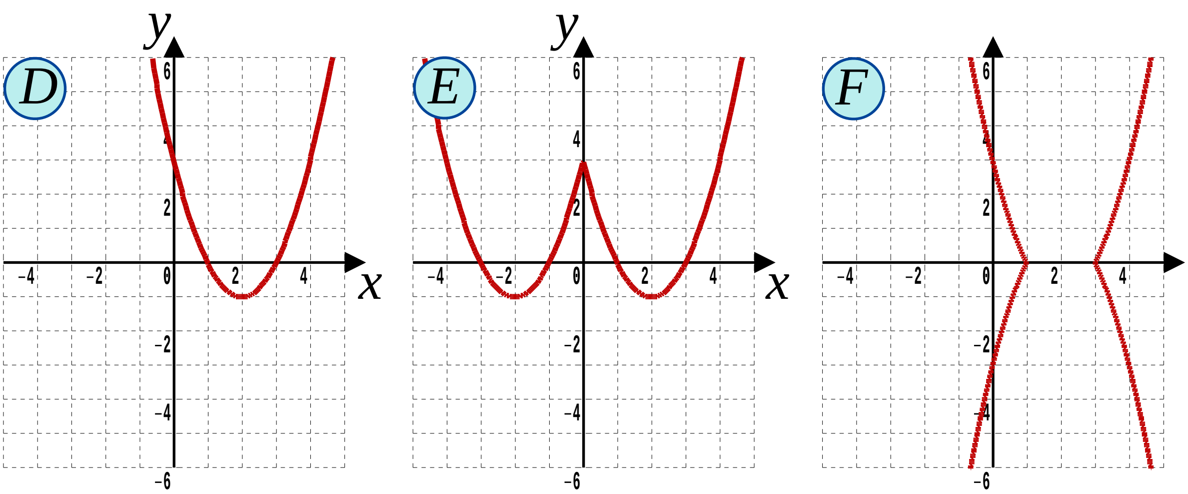
To sketch $y = f(| x |)$ **Overwrite negative x**

◇ Sketch the curve of $y = f(x)$ for $x \geqslant 0$

◇ Reflect the parts of the curve to the right of the y-axis in the y-axis.

Graph D shows the parabola with quadratic equation $y = x^2 - 4x + 3$ and graph E shows the effect of modulising each individual x. That is, $y = |x^2| - 4|x| + 3$

Notice that the information to the left of the y-axis of the original graph has been overwritten, whilst information to the right has been "duplicated" in a reflected form.



4.3.1 Example

Find the equation that will yield graph F, where the "mirror" is in the x-axis.

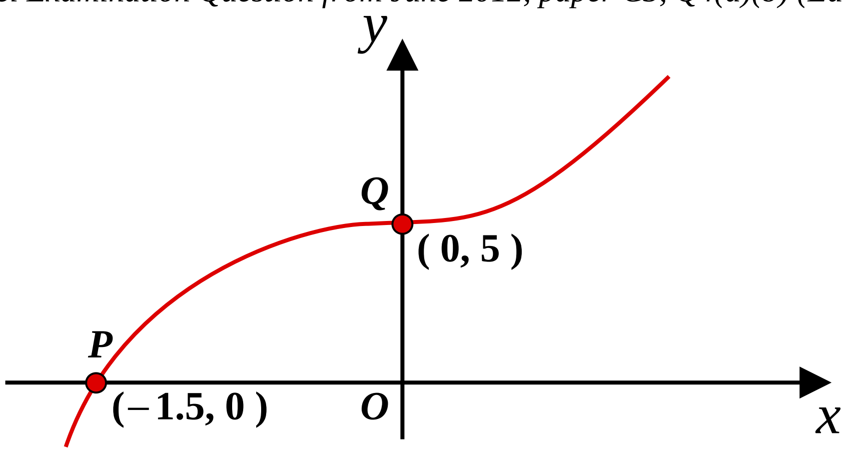
[2 marks]

4.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 100

Question 1

A-Level Examination Question from June 2012, paper C3, Q4(a)(b) (Edexcel)



The curve $y = f(x)$ passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.
On separate diagrams, sketch the curve with equation,

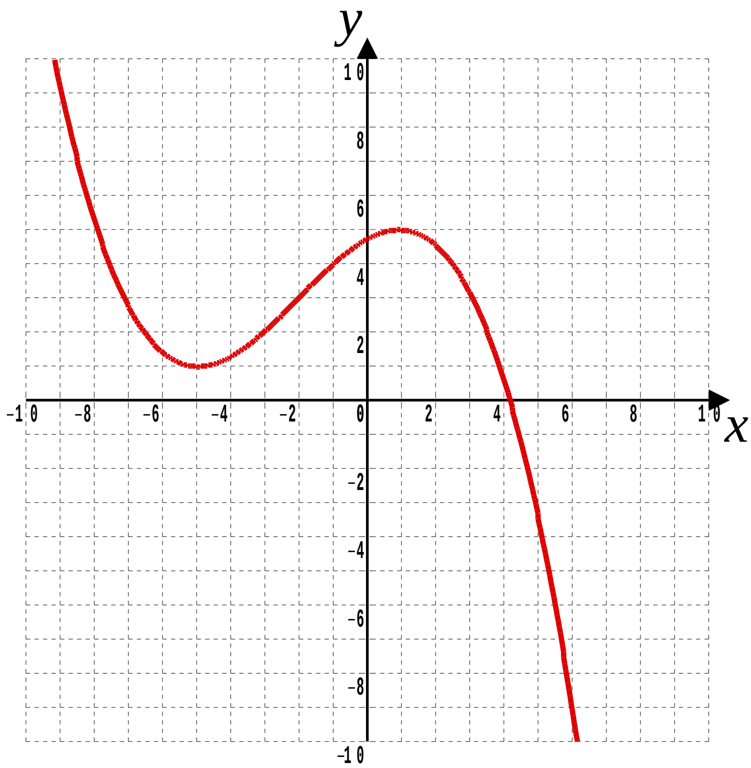
(a) $y = |f(x)|$

(b) $y = f(|x|)$

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

[2, 2 marks]

Question 2



The diagram shows the graph of $y = f(x)$

The point (1, 5) is the maximum turning point of the graph.

The point (– 5, 1) is the minimum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(i) $y = | f(x) |$ **(ii)** $y = f(| x |)$

Show on each graph the coordinates of any turning points.

[2, 3 marks]

Question 3

$$h(x) = 2(x - 3)^2 - 8, \qquad x \in \mathbb{R}$$

(a) For each of the following, draw a sketch.
Label any turning points and axes intercepts.

(i) $y = h(x)$ **(ii)** $y = |h(x)|$ **(iii)** $y = h(|x|)$

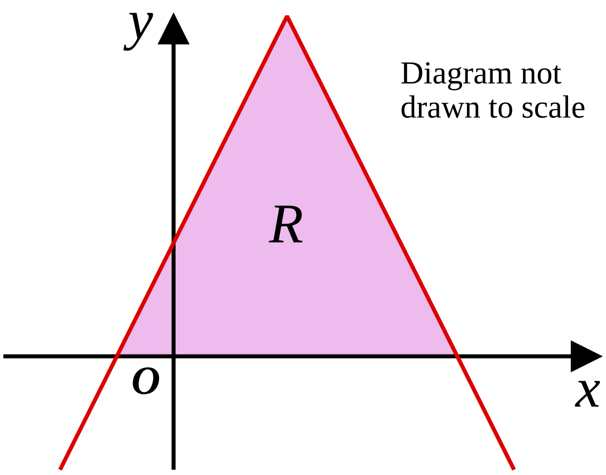
[3, 3, 3 marks]

(b) For what values of k will the equation $h(|x|) = k$ have four solutions ?

[2 marks]

Question 4

A-Level Mock Paper 2 from 2019, Q5 (Edexcel)



The graph is part of the equation $y = f(x)$ where $f(x) = 7 - |3x - 5|,\ x \in \mathbb{R}$

The finite region R, shown shaded, is bounded by the graph $y = f(x)$ and the x-axis.

(a) Find the area of R, giving your answer in its simplest form.

[4 marks]

The equation $7 - |3x - 5| = k$ where k is a constant has two distinct real solutions.

(b) Write down the range of possible values for k

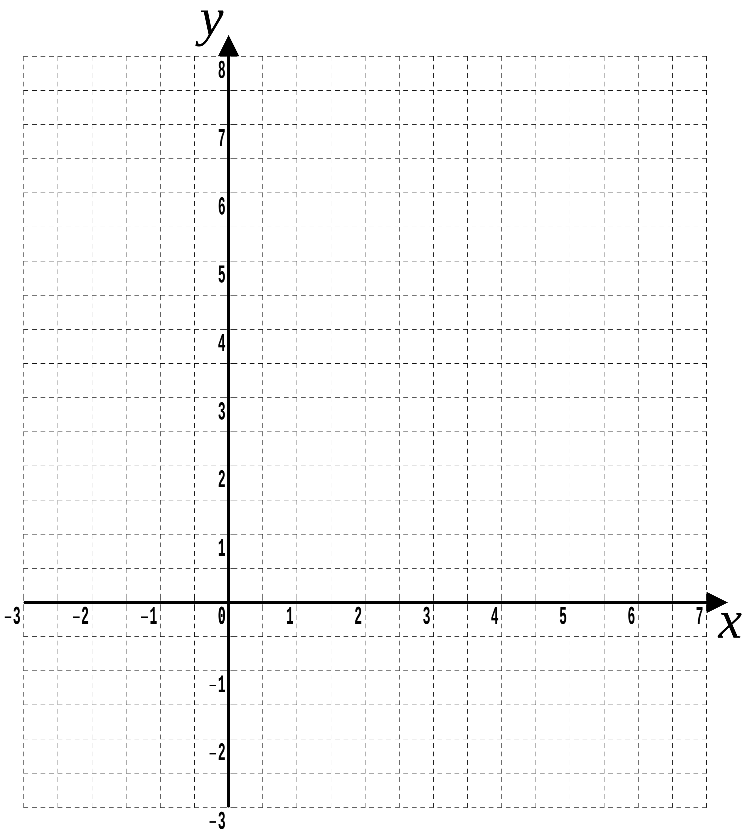
[1 mark]

Question 5

(i) Use the table below to assist in plotting the following equation;

$$|y - 3| = |x - 2|$$

x	− 3	− 2	− 1	0	1	2	3	4	5	6	7
y											



[5 marks]

(ii) Given that the following equation has exactly one real solution, find k;

$$|y - 3| - |x - 2| = k$$

[1 mark]

(iii) Given that the following equation has exactly one real solution, find c;

$$|y - 3| - |x - 2| = \frac{1}{4}x + c$$

[1 mark]

Question 6

$$f(x) = |x|^2 - 4|x| + 1 \qquad x \in \mathbb{R}$$

(a) Given that x is a real number, which of the following are true ?

(i) $|x|^2 = x^2$ **(ii)** $|x^2| = x^2$ **(iii)** $|x^2| = |x|^2$

[3 marks]

(b) Sketch the curve, $y = f(x)$, marking on the coordinates of any local minima or maxima.

[3 marks]

(c) Using algebra, solve the equation, $f(x) = -2$

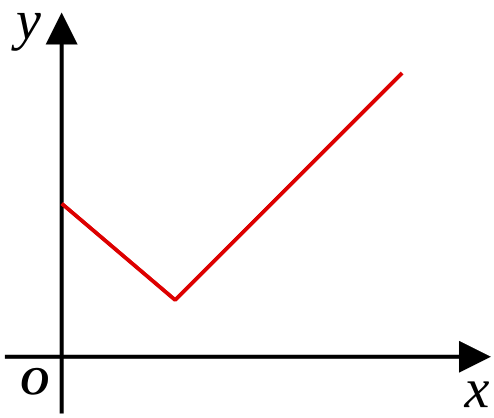
[3 marks]

(d) If $f(x) = k$, where k is a constant, has exactly two solutions, state the possible values of k

[3 marks]

Question 7

A-Level Sample Assessment Materials from 2017, Paper 2, Q11 (Edexcel)



The graph is a sketch of $y = f(x)$ where $f(x) = 2|3 - x| + 5, \quad x \geqslant 0$

(a) State the range of f

[1 mark]

(b) Solve the equation, $f(x) = \frac{1}{2}x + 30$

[3 marks]

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

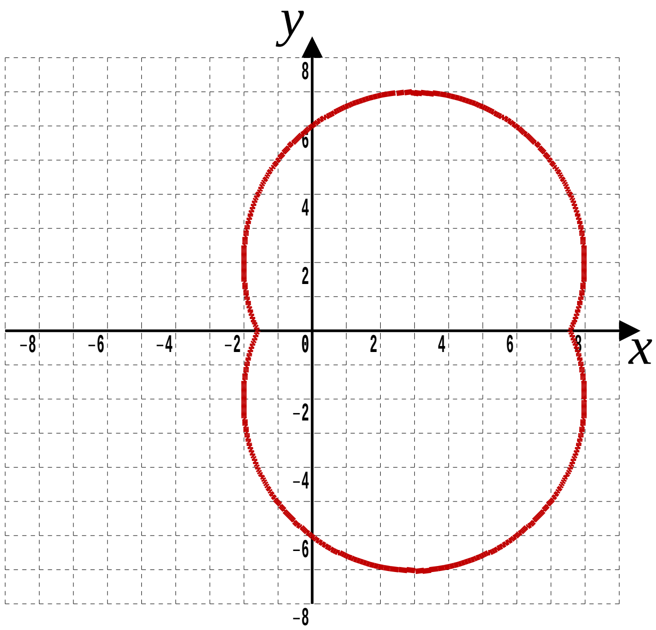
(c) state the set of possible values for k

[2 marks]

Question 8

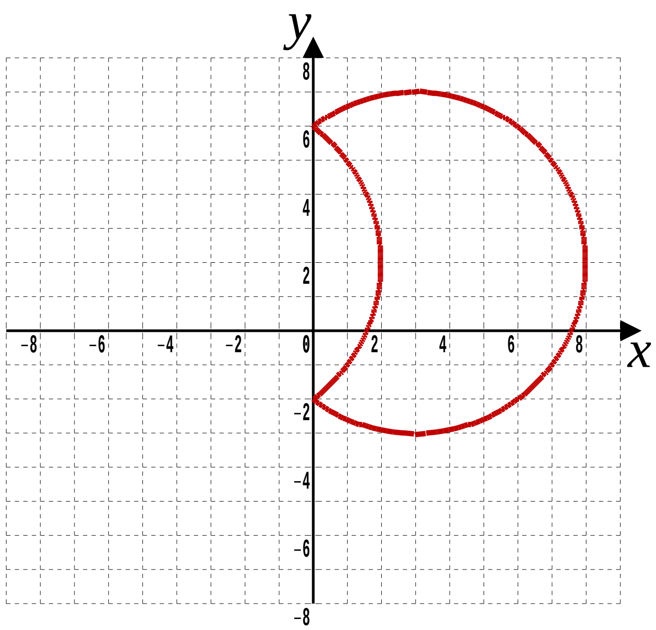
A circle centre $(3,\ 2)$ and radius 5 has equation $(x - 3)^2 + (y - 2)^2 = 25$
This equation has been manipulated in various ways and the modulus function applied to obtain the following graphs on a computer capable of processing the ± sign in an input equation.
Underneath each graph write the equation used to produce the graph.

(i)



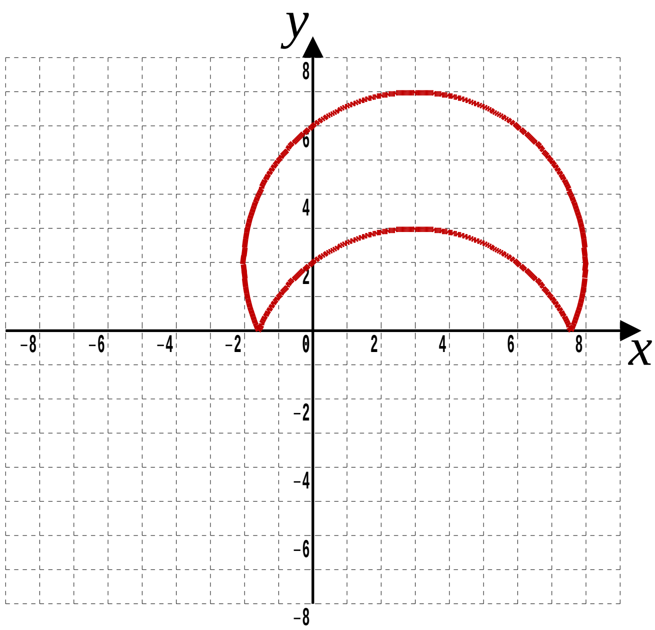
[3 marks]

(ii)



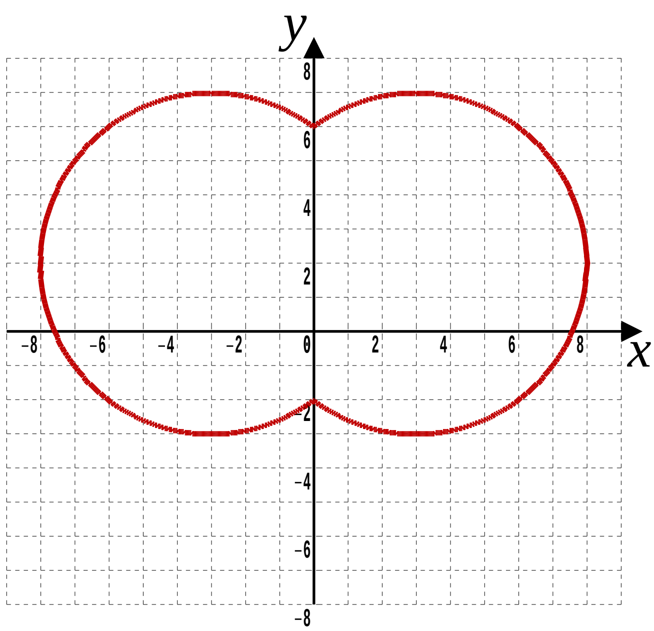
[3 marks]

(iii)



[3 marks]

(iv)



[3 marks]

If you have a graphics calculator you may like to see if you can get it to plot each of these curves. Where a ± is involved you may have to plot the equation with the plus separately to the equation with a minus.

Question 9

$$f(x) = 3x^2 - 12x + 7, \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers.

[3 marks]

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

[3 marks]

(c) **(i)** Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where,

$$g(x) = 3(x - 1)^2 - 12x + 9, \qquad x \in \mathbb{R}$$

[2 marks]

(ii) Find the range of the function

$$h(x) = \frac{20}{3x^2 - 12x + 7} \qquad x \in \mathbb{R}$$

[2 marks]

Question 10

A-Level Examination Question from January 2014, Paper C3, Q6 (Edexcel)

Given that a and b are constants and that $0 < a < b$,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x + a|$ **(ii)** $y = |2x + a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the axes.

[6 marks]

(b) Solve, for x, the equation

$$|2x + a| - b = \frac{1}{3}x$$

giving any answers in terms of a and b

[4 marks]

Question 11

The functions *f* and *g* are defined as,

$$f(x) = 4a^2 - x^2, \quad x \in \mathbb{R}$$

$$g(x) = |4x - a|, \quad x \in \mathbb{R}$$

where *a* is a constant, such that $a \geqslant 1$

(i) Sketch the graph of $f(x)$ and the graph of $g(x)$ on the same diagram.

The sketch must include the coordinates of any points where each of the graphs meets the coordinate axes.

[6 marks]

(ii) Find, in exact form where appropriate, the solutions of the equation,

$$4 - x^2 = |4x - 1|$$

[4 marks]

Question 12

Consider the function

$$f(x) = \frac{\pi}{180}x - 5 + \sin x, \qquad -300° \leqslant x \leqslant 600$$

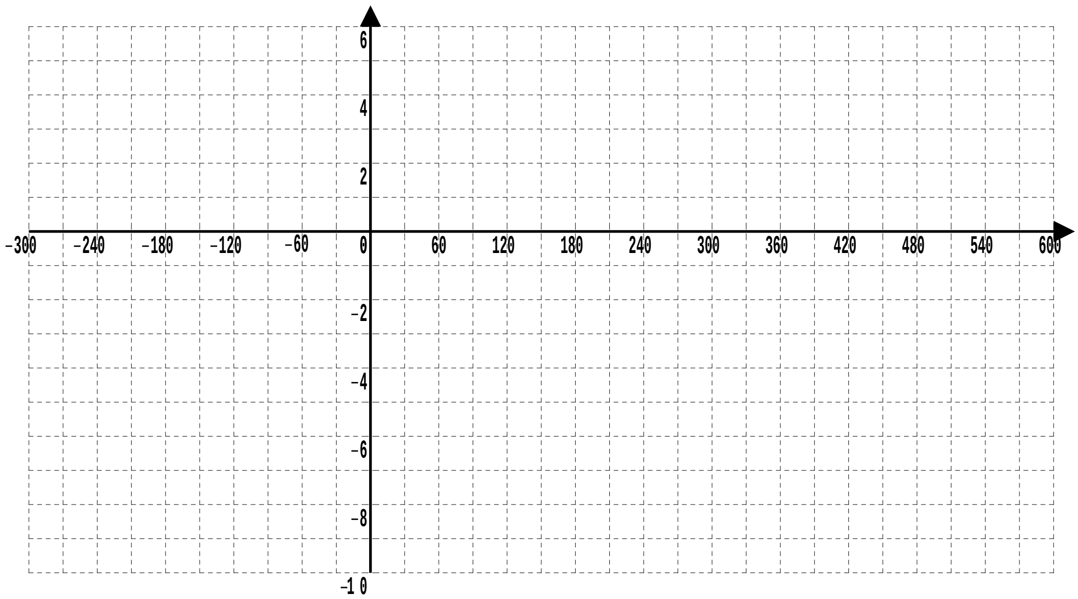
(a) Complete the following table,

x	– 300	– 240	– 180	– 120	– 60	0	60	120
y								

x	180	240	300	360	420	480	540	600
y								

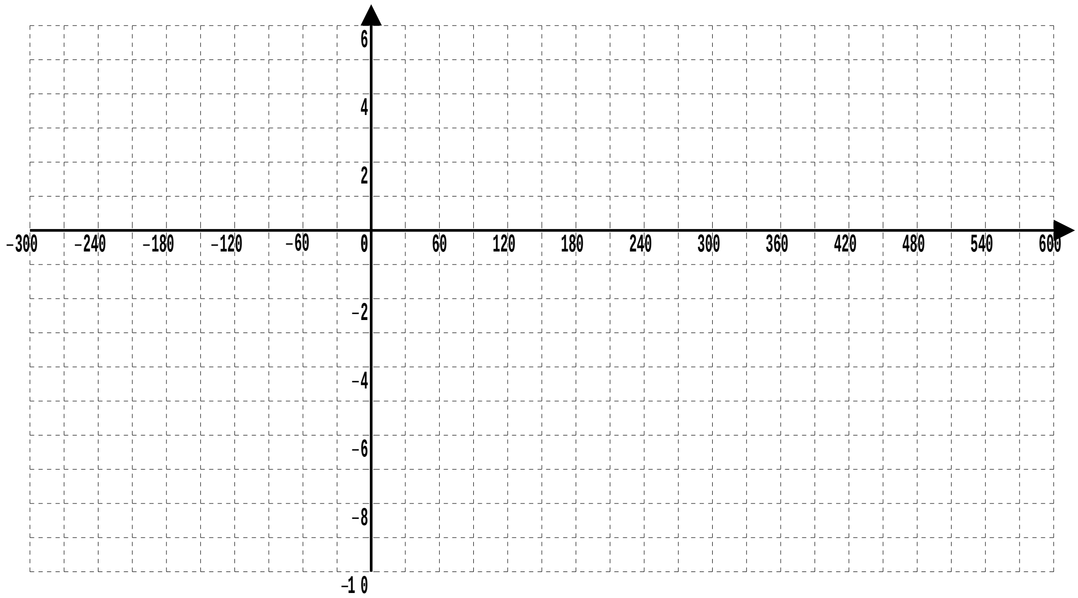
[3 marks]

(b) **(i)** On the grid below plot the graph of $y = f(x)$



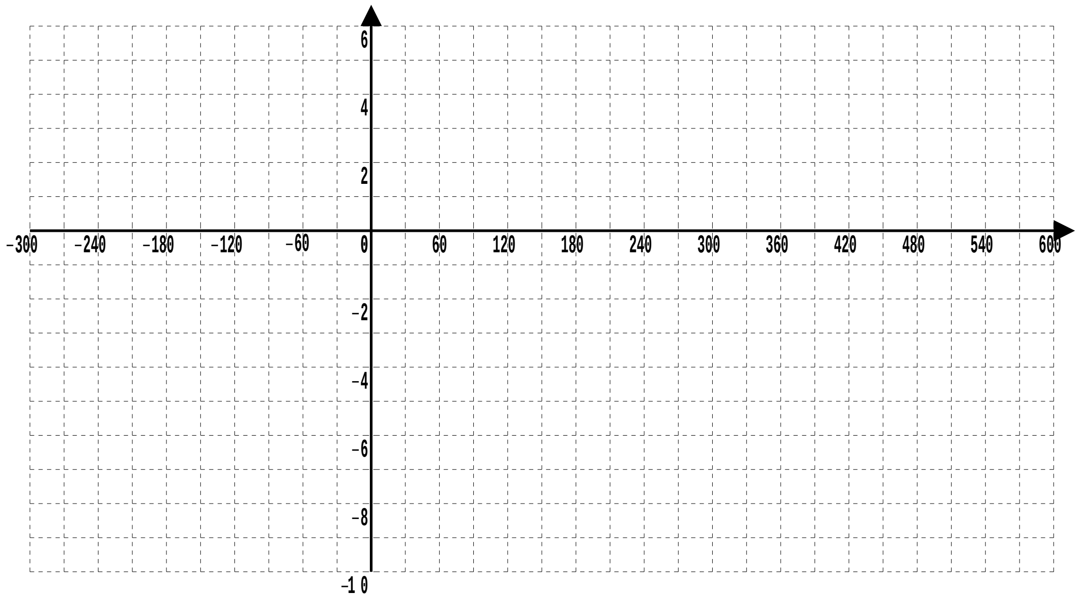
[2 marks]

(ii) On the grid below plot the graph of $y = | f(x) |$



[2 marks]

(iii) On the grid below plot the graph of $y = f(|x|)$



[2 marks]