Lesson 6

A-Level Pure Mathematics, Year 2
Functions II

6.1 Composite Functions

The topic of Composite Functions is covered at GCSE.
At that level, the emphasis is on numerical problems, with the algebraic treatment kept simple and straightforward. For A Level, the emphasis is on the algebra along with concern about domains and ranges.

Keep in mind that *fg(x)* means *eff the already gee'd x*

6.2 Example

Let *s* and *t* be the functions

$$s(x) = (x + 1)^2 \qquad x \in \mathbb{R}$$

$$t(x) = x - 2 \qquad x \in \mathbb{R}$$

(a) Determine each of the following,

(i) $st(8)$ **(ii)** $ts(8)$

(iii) $ts(x)$ **(iv)** $st(x)$

[4 marks]

(b) Find the unique value of *x* such that

$$ts(x) = st(x)$$

[2 marks]

Notice that, in general, $st(x) \neq ts(x)$

6.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 60

Question 1

Given that, $f(x) = 3x + 2, \quad x \in \mathbb{R}$ and $g(x) = 5x - 4, \quad x \in \mathbb{R}$

Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 2

Given that, $f(x) = 7x - 5, \quad x \in \mathbb{R}$ and $g(x) = 10 - x, \quad x \in \mathbb{R}$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 3

Given that, $f(x) = x^2 + x, \quad x \in \mathbb{R}$ and $g(x) = x + 3, \quad x \in \mathbb{R}$

(i) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

(ii) Find the two values of x such that, $fg(x) = 0$

[2 marks]

Question 4

Given that, $f(x) = 4x - 1, \quad x \in \mathbb{R}$ and $g(x) = x^2 + 1, \quad x \in \mathbb{R}$

Determine the following, giving answers free of any brackets;

(i) $f(3)$ (ii) $g(4)$

(iii) $fg(1)$ (iv) $gf(2)$

(v) $fg(x)$

(vi) State the range of your part **(v)** composite function

[1, 1, 2, 2, 3, 1 marks]

Question 5

$$f(x) = \frac{12}{x+5} \qquad x \in \mathbb{R},\ x \neq -5$$

$$g(x) = 6x - 5 \qquad x \in \mathbb{R}$$

Find a simplified expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 6

$$s(x) = x^2 + x, \qquad x \in \mathbb{R}$$

$$t(x) = x - 2, \qquad x \in \mathbb{R}$$

(a) Determine;

(i) $s(5)$ **(ii)** $sss(1)$ **(iii)** $ttt(1)$

(iv) $ts(10)$ **(v)** $st(3)$ **(vi)** $ts(-1)$

[3 marks]

(b) Determine $s\,t\,(x)$ writing your answer without any brackets.

[2 marks]

(c) Find the two values of x for which $st(x) = 0$

[2 marks]

Question 7

$$f(x) = \sqrt{x} + 3 \qquad x \in \mathbb{R},\ x \geqslant 0$$

$$g(x) = x + 2 \qquad x \in \mathbb{R}$$

(i) Find $fg(x)$

[2 marks]

(ii) State the domain of $fg(x)$ given that it should be as large as possible

[1 mark]

(iii) State the corresponding range of $fg(x)$

[1 mark]

Question 8

$$f(x) = x^2 - 1 \qquad x \in \mathbb{R}$$

Solve the equation,

$$ff(x) = 0$$

[4 marks]

Question 9

$$f(x) = x^2 + 8 \qquad x \in \mathbb{R}$$

$$g(x) = 2x - 5 \qquad x \in \mathbb{R}$$

Solve the equation

$$fg(x) = gf(x)$$

giving your solutions as exact surds.

[5 marks]

Question 10

Let two functions, *m* and *n*, be;

$$m(x) = 10x - 9, \qquad x \in \mathbb{R}, \qquad x \geqslant 0.9$$

$$n(x) = 100 - \sqrt{x} \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 9820.81$$

Determine each of the following, giving bracket free answers;

(i) $n(64)$

(ii) $m(3)$

(iii) $mn(9)$

(iv) $nm(9)$

(v) $mn(x)$

(vi) $nm(x)$

[8 marks]

(vii) Function *m* is only defined on domain $x \geqslant 0.9$

To see why, calculate $m(0)$ then $nm(0)$

[2 marks]

(viii) Having had to restrict the domain of *m* so that *nm* exists, the domain of *n* has to then also be restricted so that its output can be fed into *m*. Calculate, *n*(9820.8) and explain why any input greater than the 9820.81 would cause a problem.

[2 marks]

Observation : *fg*, can be formed only if the range of *g* is a subset of the domain of *f*.

Question 11

Let m and n be the functions;

$$m(x) = 9x - 5 \qquad x \in \mathbb{R}$$

$$n(x) = \sqrt{x - 7} \qquad x \in \mathbb{R},\ x \geqslant 7,$$

Evaluate each of the following;

(i) $mn(8)$

(ii) $mn(56)$

(iii) $mn\left(z^2 + 7\right)$

(iv) $mn\left(4z^2 + 7\right)$

[8 marks]