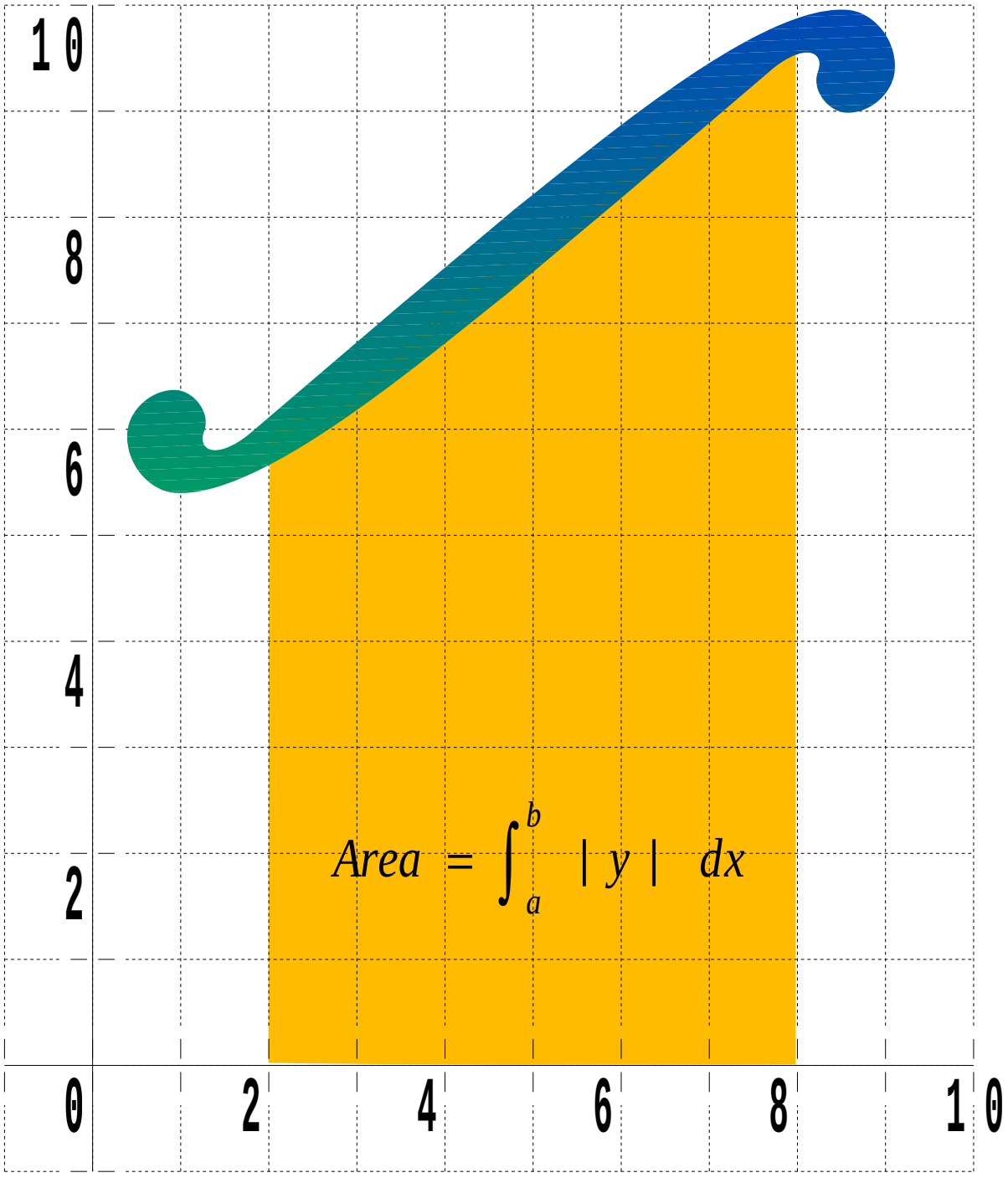
INTEGRATION



A-Level Pure Mathematics (Year 1)
Additional Mathematics

I N T E G R A T I O N I

Lesson 1

A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

1.1 The Fundamental Theorem Of Calculus

In the early 1600s there were two big unsolved problems in mathematics to do with curves.

- The first was to find the gradient of a curve
- The second was to find the area under a curve

It came as a stunning revelation to find that these two problems were different aspects of the same mathematical process. Each was the reverse of the other.
To emphasise the importance of this connection it is known under the grand title of;

The Fundamental Theorem of Calculus

The process of integration (finding areas under curves)
is the inverse of
the process of differentiation (finding gradients of curves)

1.2 Backwards Differentiation

Generally speaking, the process of differentiation is easier than integration and often an integration answer is checked by differentiation to make sure that the integration answer is then turned back into its question.
The following rule is key,

Definite Integration Power Rule

$$\text{If} \quad y = x^n \quad \text{then} \quad \int_a^b y \, dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

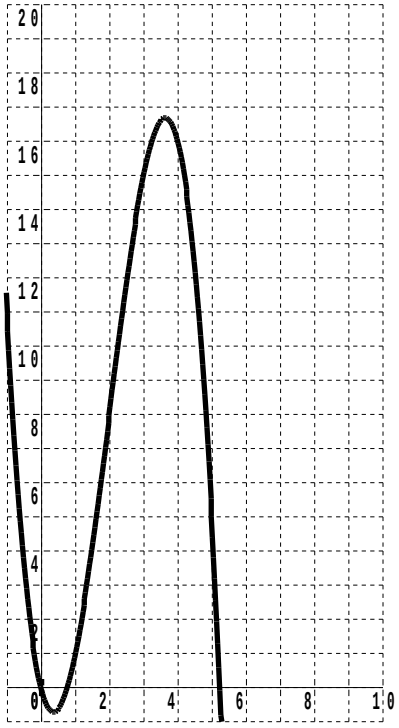
1.3 A First Integration Example

Teaching Video : http://www.NumberWonder.co.uk/v9043/1.mp4

<= The video will talk through the example on the next page

Find the area between the curve with equation $y = 6x^2 - x^3 - 4x$ and the x-axis from $x = 1$ to $x = 4$

Begin by shading in the area to be found on the following graph;



Translated into mathematics, the question becomes;

$$Area = \int_1^4 6x^2 - x^3 - 4x \; dx$$

$$= \left[\frac{6x^3}{3} - \frac{x^4}{4} - \frac{4x^2}{2} \right]_1^4$$

$$= \left[2x^3 - \frac{x^4}{4} - 2x^2 \right]_1^4$$

$$= \left[2 \times 4^3 - \frac{4^4}{4} - 2 \times 4^2 \right] - \left[2 \times 1^3 - \frac{1^4}{4} - 2 \times 1^2 \right]$$

$$= [\, 128 - 64 - 32 \,] - \left[2 - \frac{1}{4} - 2 \right]$$

$$= [\, 32 \,] - \left[-\frac{1}{4} \right]$$

$$= 32.25$$

1.4 Definite and Indefinite Integration

◊ ***Definite*** Integration questions are those that involving limits.
A calculator can be used to check the numerical answer.

◊ ***Indefinite*** Integrations are the algebra part only, without any limits.

1.5 Integrating a Isolated Number

An isolated number is a number that is not multiplying any *x*.
When integrated it integrates to that number of *x*.
So, for example, suppose the following isolated 1 is to be integrated,

$$\int_3^7 1 \; dx$$

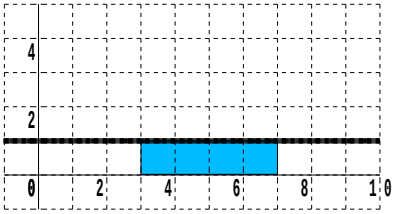
☞

It integrates to 1*x* or just *x*

$$\int_3^7 1 \; dx = [\, x \,]_3^7$$

$$= [\, 7 \,] - [\, 3 \,]$$

$$= 4$$



The area graph is a rectangle of height 1 and width 4 which obviously has an area of 4

1.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Show that the following integral has a value of 237

$$\int_2^5 6x^2 + 1 \; dx$$

[3 marks]

Question 2

Find the value of,

$$\int_1^3 24x^3 - 6x \; dx$$

[3 marks]

Question 3

Find the area between $y = 36x^2$ and the x-axis from $x = 3$ to $x = 5$

[3 marks]

Question 4

Show that the following integral has a value of 3.2

$$\int_1^2 x^4 - 3 \; dx$$

[3 marks]

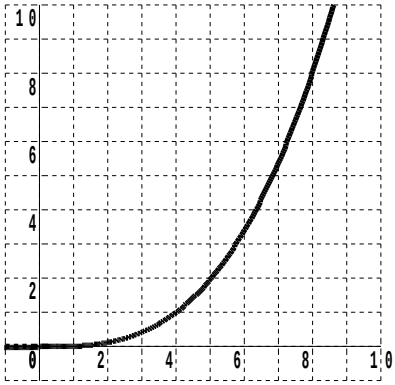
Question 5

This question is about finding the area between the curve

$$y = \frac{x^3}{64}$$

and the x-axis from $x = 4$ to $x = 8$

(i) Shade in the area to be found on the following graph of the curve;



[1 mark]

(ii) Show by integration that the area specified is 15 exactly

[3 marks]

Question 6

By first expanding the brackets, evaluate,

$$\int_2^3 (3x + 2)(x + 1)\, dx$$

[4 marks]

Question 7

Additional Mathematics Examination Question from June 2006, Q1 (OCR)

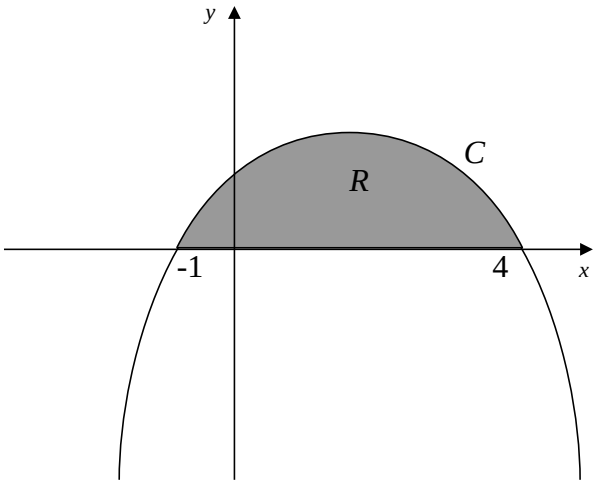
Find

$$\int_1^3 (x^2 + 3)\, dx$$

[4 marks]

Question 8

A-Level Examination Question from January 2009, Paper C2, Q2 (Edexcel)



The diagram shows part of the curve C with equation $y = (1 + x) (4 – x)$

The curve intersects the x-axis at $x = – 1$ and $x = 4$
The region R, shown shaded, is bounded by C and the x-axis.

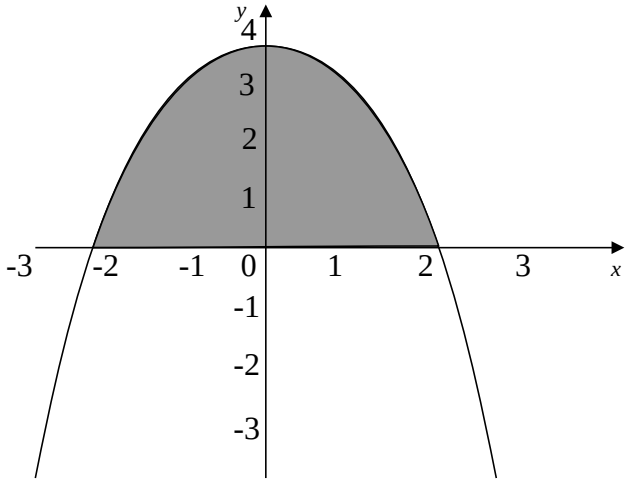
Use calculus to find the exact area of R.

[5 marks]

Question 9

Additional Mathematics Examination Question from June 2004, Q2 (OCR)

The curve shown is part of the graph of $y = 4 - x^2$

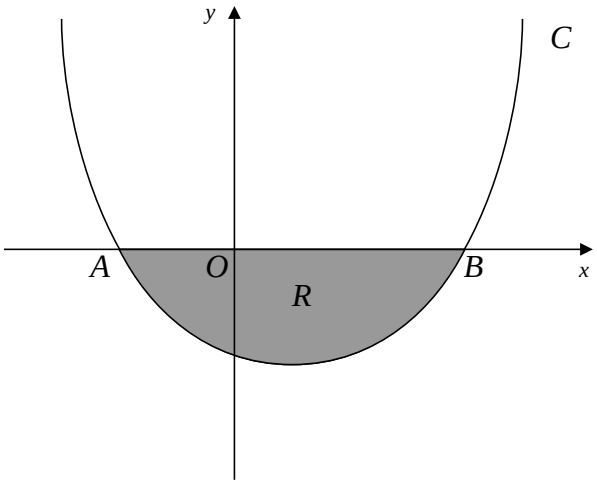


Calculate the area of the shaded region between this curve and the *x*-axis, giving your answer as an exact fraction.

[4 marks]

Question 10

A-Level Examination Question from January 2011, Paper C2, Q4 (Edexcel)



The diagram shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

[1 mark]

The finite region R, shown shaded, is bounded by C and the x-axis.

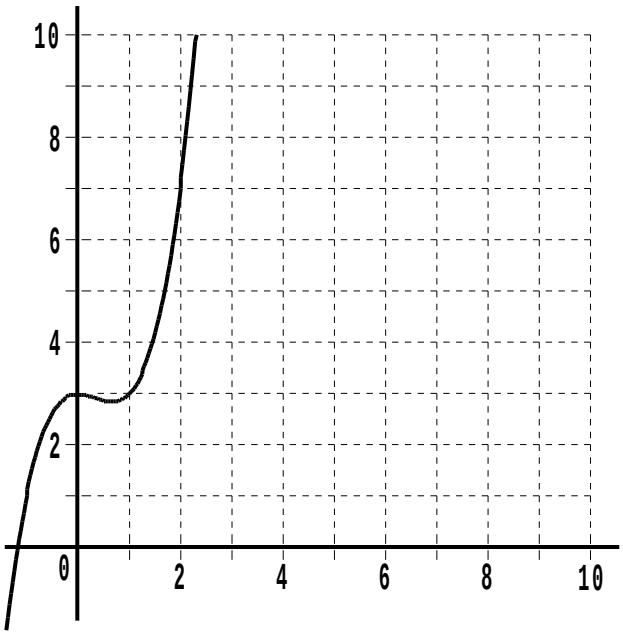
(b) Use integration to find the area of R.

[6 marks]

Question 11

Find the area between $y = x^3 - x^2 + 3$ and the x-axis from $x = 1$ to $x = 2$

(i) Begin by shading in the area to be found on the following graph.



[1 mark]

(ii) Find by integration the exact area of the specified region.

[4 marks]

Question 12

Show that,

$$\int_1^5 \frac{1}{x^3}\, dx = \frac{12}{25}$$

HINT : $\frac{1}{x^3} = x^{-3}$

[5 marks]