Lesson 3

3.1 Integration with Fractional Powers

Fractional powers arise quite naturally in integration questions, not least because

$$\sqrt{x} \;=\; x^{\frac{1}{2}}$$

Although the rule to integrate expressions containing such powers is no different to that for integrating integer powers, there is a skill in avoiding multi-decker fractions, as the next example will demonstrate.

3.2 "Fractional Power" Example

The curve graphed is of the equation $y \;=\; x^{\frac{3}{4}}$

Determine the shaded area

Teaching Video : http://www.NumberWonder.co.uk/v9043/3.mp4
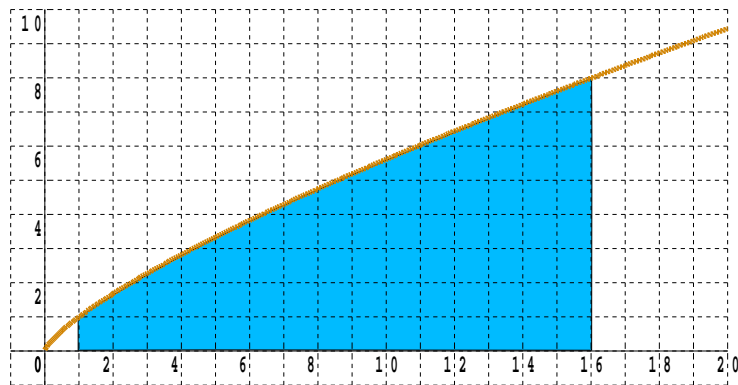
[4 marks]

3.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Show that $\displaystyle\int_{4}^{25} \sqrt{x}\ dx\ =\ 78$

There is a bonus mark for not having any multi-decker fractions in your working !

[4 marks, plus 1 bonus mark]

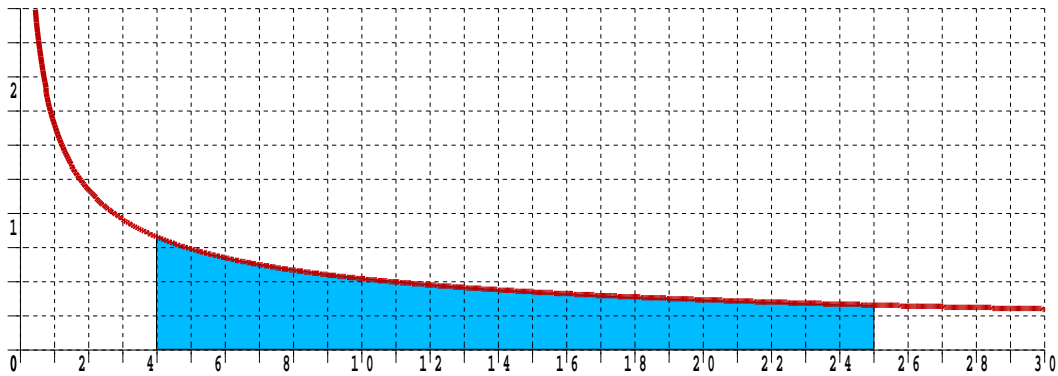### Question 2

Determine the exact value of $\displaystyle\int_{0}^{3} \sqrt{3x}\ dx$

HINT : $\displaystyle\int_{0}^{3} \sqrt{3x}\ dx\ =\ \sqrt{3}\ \int_{0}^{3} \sqrt{x}\ dx$

[4 marks]

Question 3

The graph shows the curve with equation $y = \dfrac{5}{3\sqrt{x}}$

Determine the exact value of $\displaystyle\int_{4}^{25} \dfrac{5}{3\sqrt{x}}\ dx$

HINT : $\dfrac{5}{3\,x^{\frac{1}{2}}} = \dfrac{5\,x^{-\frac{1}{2}}}{3}$

[5 marks]

Question 4

Given that $y = \dfrac{\sqrt{x} + 1}{3}$ find in the simplest form $\displaystyle\int_1^4 y \, dx$

HINT : $\displaystyle\int_1^4 \dfrac{\sqrt{x} + 1}{3} \, dx = \dfrac{1}{3} \int_1^4 \sqrt{x} + 1 \, dx$

[5 marks]

Question 5

A-Level Examination Question from June 2009, Paper C2, Q1 (Edexcel)
Use calculus to find the exact value of

$$\int_1^4 \left(2x + 3\sqrt{x} \right) dx$$

[5 marks]

Question 6

A-Level Examination Question from January 2017, Paper C12, Q7(ii) (Edexcel)

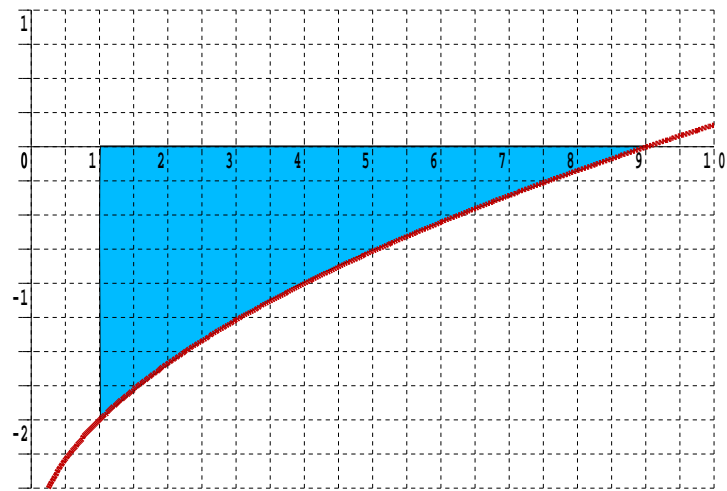Given that k is a constant and

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k \right) dx = 30$$

find the exact value of k

[6 marks]

Question 7

The curve graphed below is of the equation $y = \sqrt{x} - 3$

Taking great care over the minus signs in the working, determine the shaded area.

[6 marks]

Question 8
A-Level Examination Question from May 2007, Paper C2, Q1 (Edexcel)
Evaluate

$$\int_1^8 \frac{1}{\sqrt{x}}\, dx$$

giving your answer in the form $a + b\sqrt{2}$, where a and b are integers

[4 marks]