6.1 The Constant Of Integration

Last lesson made use of the following rule,

Indefinite Integration Power Rule

$$\text{If} \quad y = x^n \quad \text{then} \quad \int y \, dx = \frac{x^{n+1}}{n+1} + c$$

Proof

Let $f(x) = \dfrac{x^{n+1}}{n+1} + c$ where c is a constant

in which case the derivative is given by,

$$f'(x) = \frac{(n+1)\,x^{n+1-1}}{n+1}$$
$$= x^n$$

As the derivative of $\dfrac{x^{n+1}}{n+1} + c$ is x^n then, by the fundamental theorem of calculus, the integral of x^n must be $\dfrac{x^{n+1}}{n+1} + c$ □

A logical question at this point would be to ask why c, often called "the constant of integration" was ignored when considering integrations with limits;

Definite Integration Power Rule

$$\text{If} \quad y = x^n \quad \text{then} \quad \int_a^b y \, dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

Here is the answer :

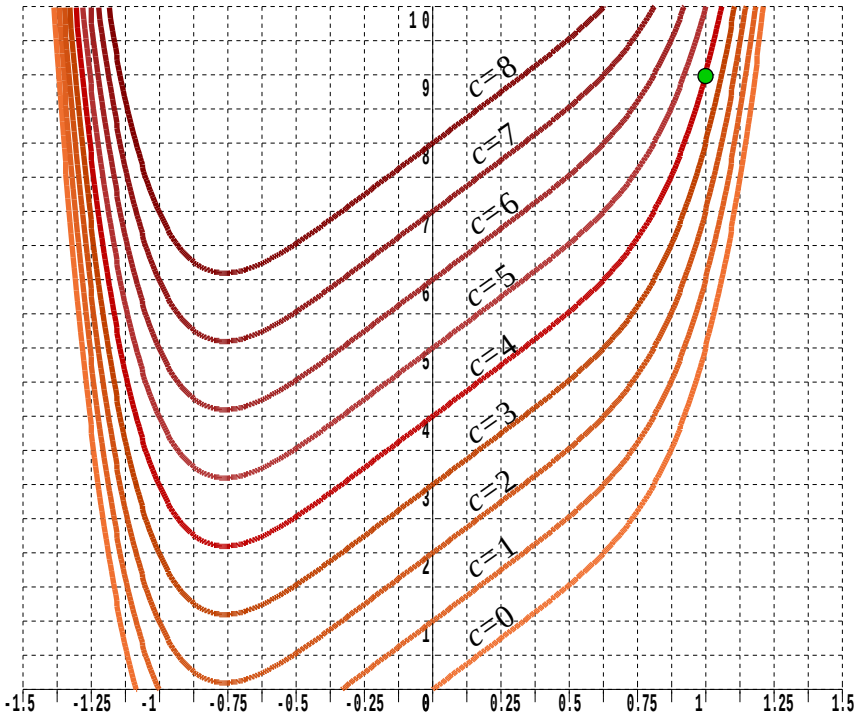
$$\left[\frac{x^{n+1}}{n+1} + c \right]_a^b = \left[\frac{b^{n+1}}{n+1} + c \right] - \left[\frac{a^{n+1}}{n+1} + c \right]$$
$$= \left[\frac{b^{n+1}}{n+1} \right] + c - \left[\frac{a^{n+1}}{n+1} \right] - c$$
$$= \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

6.2 Example

The gradient function of a curve is given by $\frac{dy}{dx} = 12x^5 + 3$

Find the equation of the curve given that it passes through the point $(1, 9)$

Teaching Video : http://www.NumberWonder.co.uk/v9043/6.mp4



All these curves have gradient equation $\frac{dy}{dx} = 12x^5 + 3$

but only one passes through the point $(1, 9)$

6.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 40

Question 1

Additional Mathematics Examination Question from June 2011, Q9 (OCR)

The gradient function of a curve is given by;

$$\frac{dy}{dx} = 3x^2 - 2x + 4$$

Find the equation of the curve given that it passes through the point (2, 2)

[4 marks]

Question 2

Additional Mathematics Examination Question from June 2018, Q2 (OCR)

The gradient function of a curve is given by;

$$\frac{dy}{dx} = 2 + 2x - 3x^2$$

Find the equation of the curve given that it passes through the point (2, 3)

[4 marks]

Question 3

A-Level Examination Question from January 2012, Paper C1, Q7 (Edexcel)

A curve with equation $y = f(x)$ passes through the point $(2, 10)$

Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of $f(1)$

[5 marks]

Question 4

A-Level Examination Question from January 2018, Paper C12, Q1 (Edexcel)

Given that $y = \frac{2x^{\frac{2}{3}} + 3}{6}$, $x > 0$ find, in the simplest form,

(a) $\frac{dy}{dx}$

[2 marks]

(b) $\int y \, dx$

[3 marks]

Question 5

A-Level Examination Question from January 2017, Paper C12, Q7(i) (Edexcel)

Find $\int \frac{2 + 4x^3}{x^2}\, dx$ giving each term in its simplest form

[4 marks]

Question 6

A-Level Examination Question from January 2013, Paper C1, Q8 (Edexcel)

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \qquad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form

[6 marks]

Question 7

A-Level Examination Question from January 2014, Paper C1, Q9 (Edexcel)

A curve with equation $y = f(x)$ passes through the point $(3, 6)$

Given that $f'(x) = (x - 2)(3x + 4)$

(a) use integration to find $f(x)$

Give your answer as a polynomial in its simplest form

[5 marks]

(b) Show that $f(x) = (x - 2)^2(x + p)$, where p is a positive constant.

State the value of p

[3 marks]

(c) Sketch the graph of $y = f(x)$, showing the coordinates of any points where the curve touches or crosses the coordinate axes.

[4 marks]