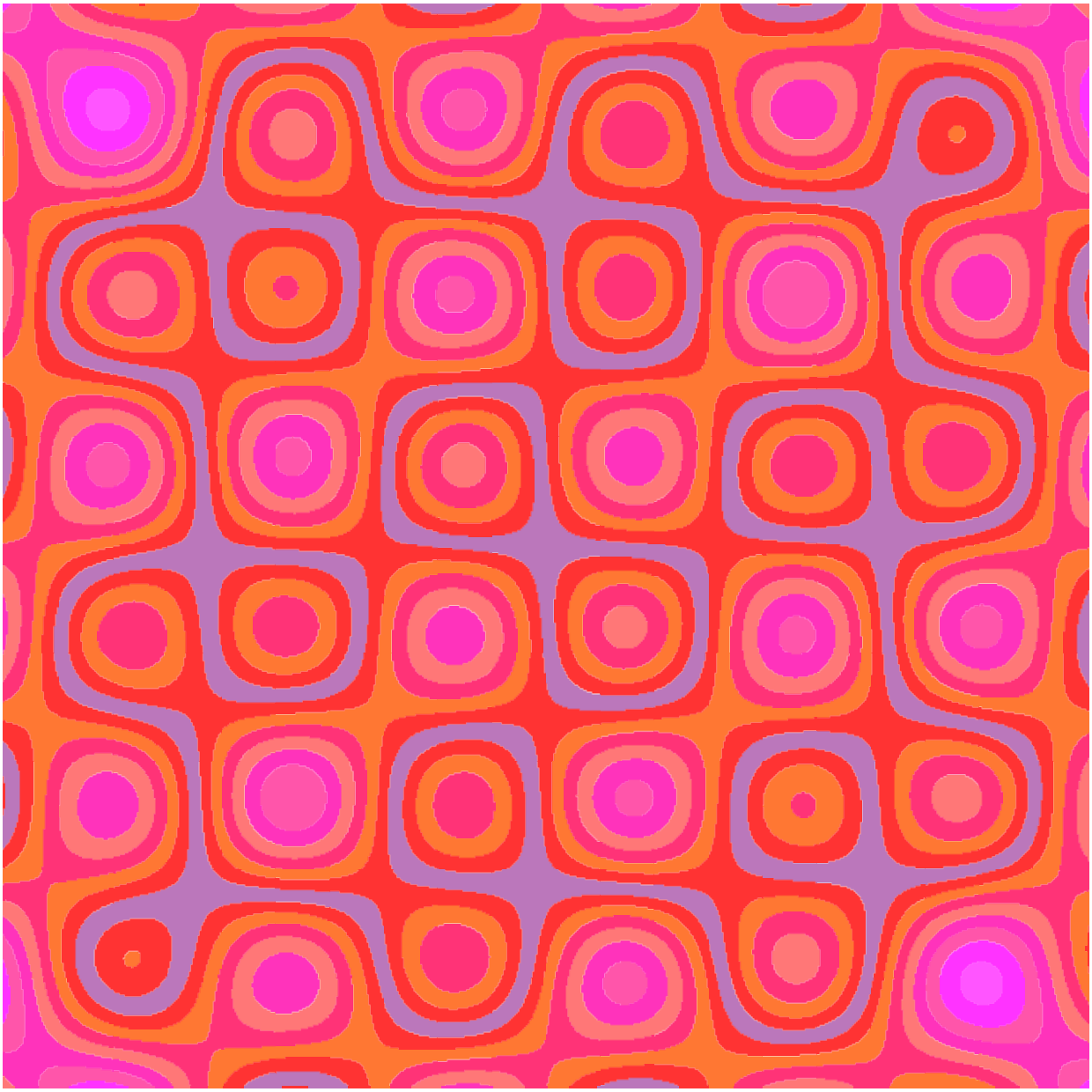
Trigonometry IV

Additional Mathematics
A-Level Pure Mathematics : Year 1

Equations and Identities



Art from the Trigonometric Equation

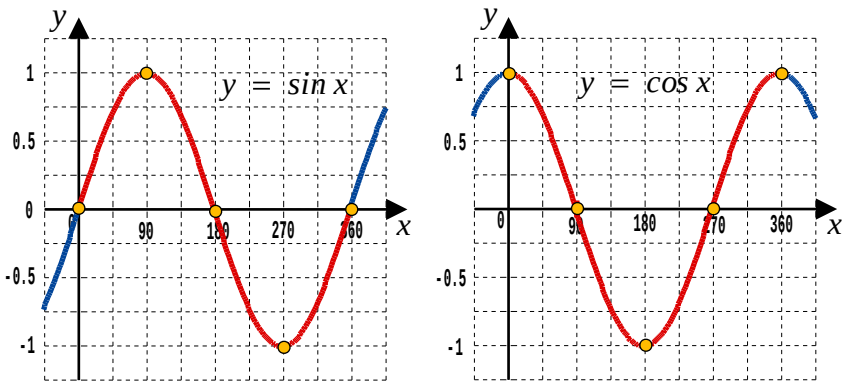
$$\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)+\left(\frac{y}{2}\right)\sin\left(\frac{y}{2}\right)+8\sin x\sin y=0$$

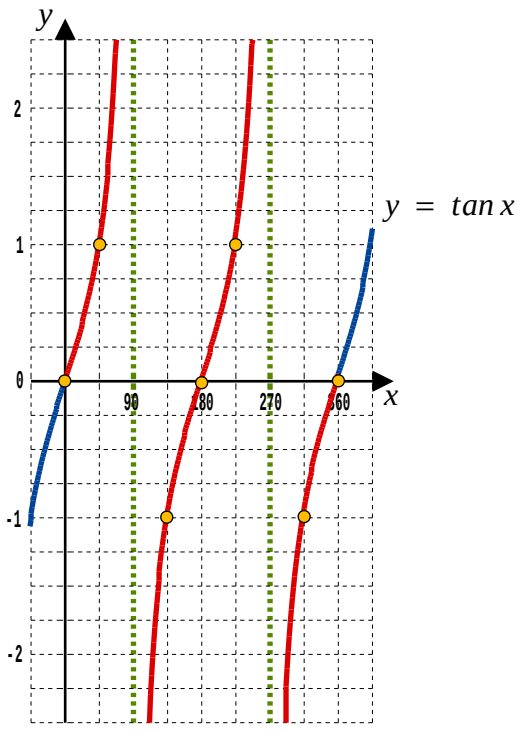
Lesson 1

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

1.1 Graph Assisted Equation Solving

To solve equations that involve *sin*, *cos* and *tan* a calculator and, from memory, a sketch of the appropriate graph is required. Here are the graphs needed "in mind".





Teaching Video : http://www.NumberWonder.co.uk/v9044/1a.mp4

← Watch the video which talks through these essential graphs

1.2 Example

The Question : Solve the equation $\cos x = -0.283 \quad 0° \leqslant x \leqslant 360°$

The Solution : It's tempting to start by using your calculator to get that

$$x = \arccos(-0.283)$$
$$= 106.4°$$

This is a solution, but it's not ALL of the solutions. Far better to begin by always getting the acute working angle by initially ignoring the minus sign.

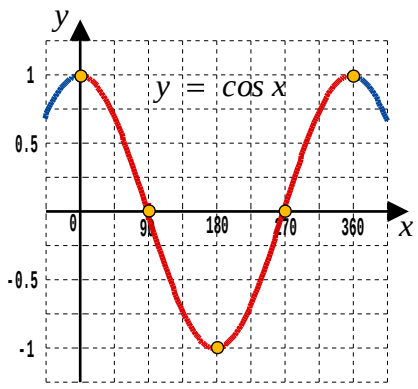
Thus; $\cos x = 0.283$

$$x = \arccos 0.283$$
$$x = 73.6° \leftarrow \text{The working (or principle) angle}$$

The solutions sought in the range $0° \leqslant x \leqslant 360°$ will be two of

$x \qquad 180 - x \qquad 180 + x \qquad$ or $\qquad 360 - x$

Now sketch the appropriate graph, either for *sin*, *cos* or *tan*
In this case we want $y = \cos x$



From the sketch graph, solution sought are 180 – 73.6 and 180 + 73.6

$$\therefore x = 106.4°, \quad 253.6°$$

[6 marks]

Teaching Video : http://www.NumberWonder.co.uk/v9044/1b.mp4

← Watch the Video which talks through the example's solution

1.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 30

Question 1

(a) Find both solutions to the equation $\cos x = 0.417$ for $0° \leqslant x \leqslant 360°$
Your solution should include a sketch of the graph of $y = \cos x$

[4 marks]

(b) Find both solutions to the equation $\cos x = -0.728$ for $0° \leqslant x \leqslant 360°$

[4 marks]

Question 2

(i) Sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$

[2 marks]

(ii) With the help of your part (i) sketch, find all solutions in the interval $0° \leqslant x \leqslant 360°$ of the following equations;

(a) $\sin x = 0.622$

[2 marks]

(b) $\sin x = -0.383$

[2 marks]

(c) $5 \sin x = 3.445$

[2 marks]

Question 3

(i) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$

[2 marks]

(ii) With the help of your part (i) sketch, find all solutions in the interval $0° \leqslant x \leqslant 360°$ of the following equations;

(a) $\tan x = 4.718$

[2 marks]

(b) $\tan x = -1.383$

[2 marks]

(c) $11 \tan x = 8$

[2 marks]

Question 4

Additional Mathematics Examination Question from June 2007, Q4 (OCR)

Find all the values of x in the range $0° < x < 360°$ that satisfy

$$\sin x = -4 \cos x$$

Hint : Divide both sides by $\cos x$. It is OK to do this because $\cos x = 0$ is not a solution so we're not dividing by zero. Then use the fact $\frac{\sin x}{\cos x} = \tan x$.

[6 marks]