Lesson 2

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

2.1 In Disguise

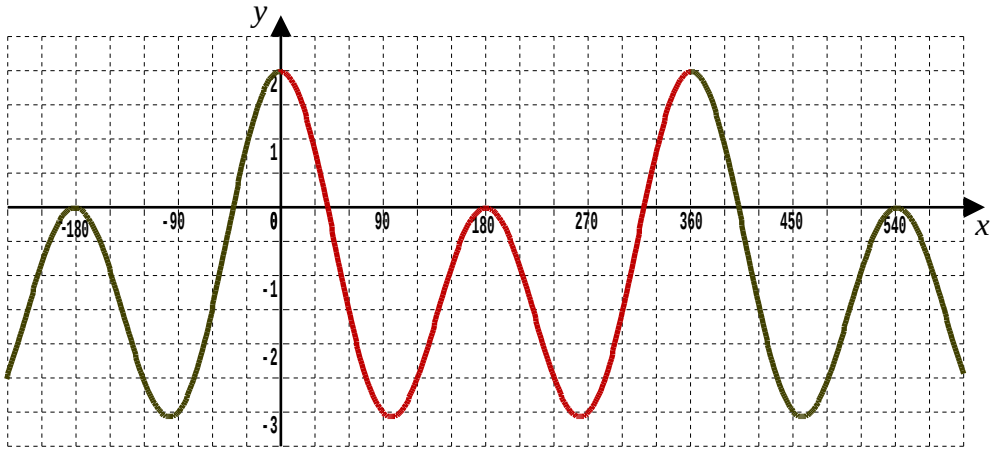
Faced with an unfamiliar mathematical situation, a technique frequently deployed is to seek a way to transform it into a more familiar situation for which a method of solution is already known.

Here is a seemingly more complicated trigonometric equation to solve;

$$4\cos^2 x + \cos x - 3 = 0 \qquad \text{for} \quad 0° \leqslant x \leqslant 360°$$

How might this be handled ?

Using a graphics calculator or a graph plotter would give some idea of the situation in which the problem is embedded.

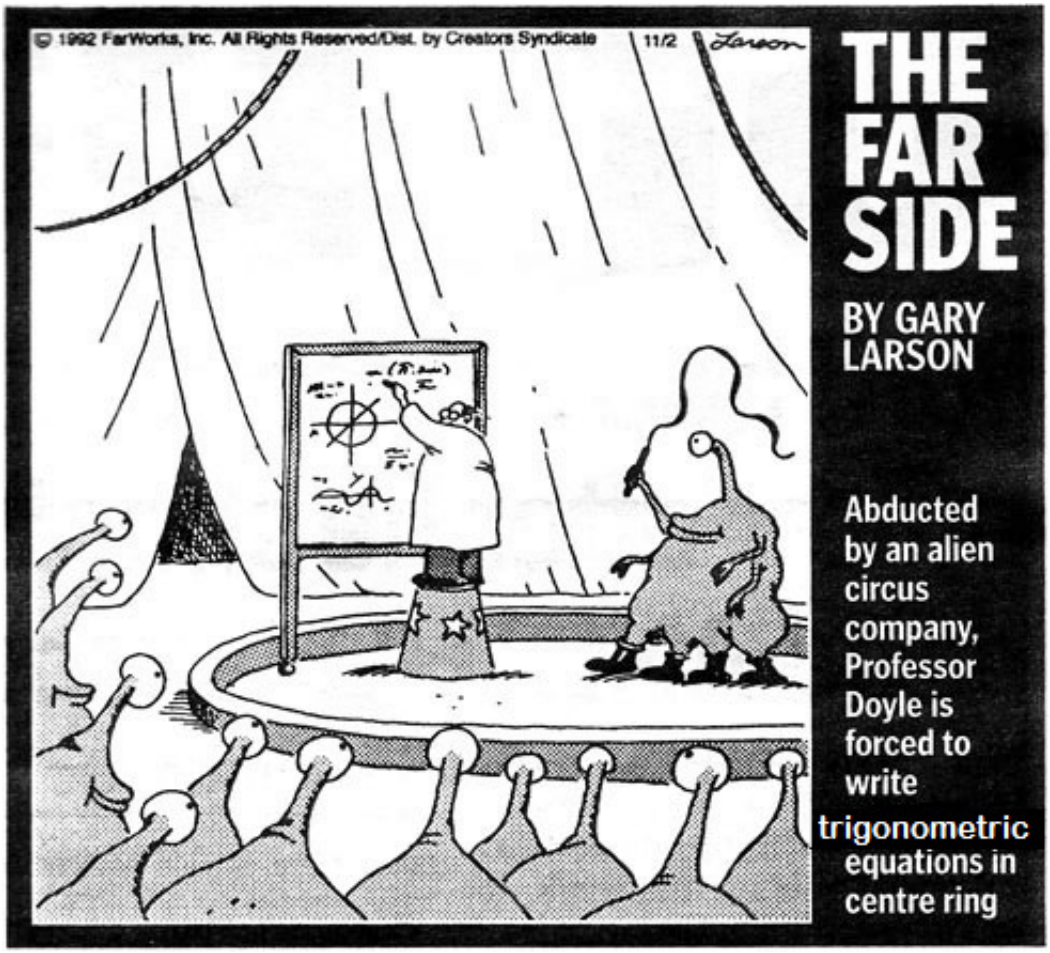


Where does this (red piece of) graph have zero height ?
It would seem that there is one solution around 45°, another around 315°, and possibly one or two more close to 180° although it's not clear if the curve gets to or crosses the x-axis near 180°.

Of course, what is sought is a mathematical method of solving the trigonometric equation which the teaching video will now reveal,

Teaching Video : http://www.NumberWonder.co.uk/v9044/2.mp4

[6 marks]



2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

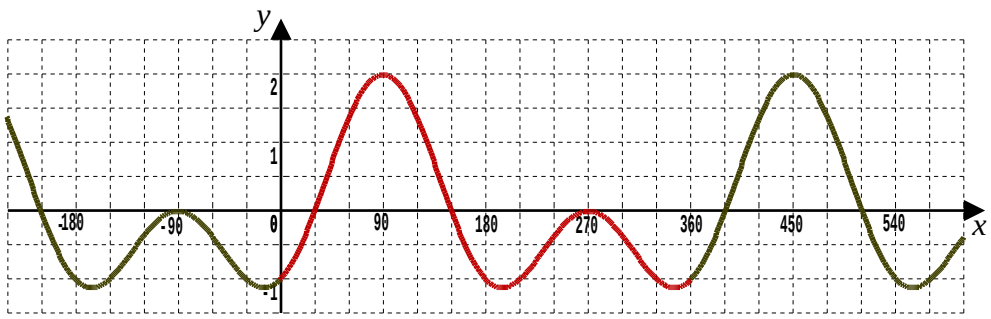
Marks Available : 30

Question 1

The graph is of the trigonometric equation,

$$f(x) = 2\sin^2 x + \sin x - 1$$

with a focus on the red piece on the interval $0° \leqslant x \leqslant 360°$



(i) From looking at the graph, write down the possible approximate values of x that would satisfy the equation

$$2\sin^2 x + \sin x - 1 = 0, \qquad 0° \leqslant x \leqslant 360°$$

[4 marks]

(ii) Use the mathematics of a *quadratic in disguise* to solve the equation

$$2\sin^2 x + \sin x - 1 = 0, \qquad 0° \leqslant x \leqslant 360°$$

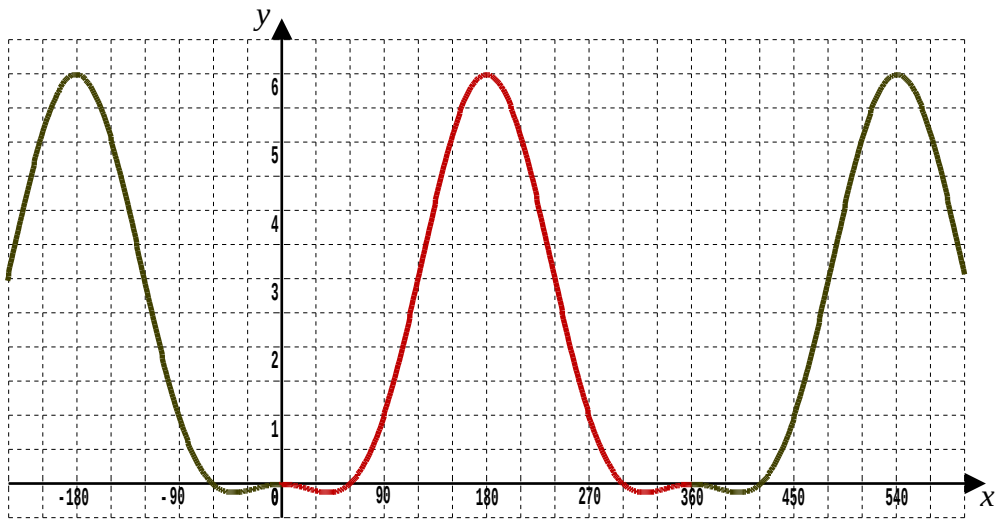
[6 marks]

Question 2

The graph is of the trigonometric equation,

$$f(x) = 2\cos^2 x - 3\cos x + 1$$

with a focus on the red piece on the interval $0° \leqslant x \leqslant 360°$



(i) From looking at the graph, write down the four approximate values of x that would satisfy the equation

$$2\cos^2 x - 3\cos x + 1 = 0, \qquad 0° \leqslant x \leqslant 360°$$

[4 marks]

(ii) Use the mathematics of a *quadratic in disguise* to solve the equation

$$2\cos^2 x - 3\cos x + 1 = 0, \qquad 0° \leqslant x \leqslant 360°$$

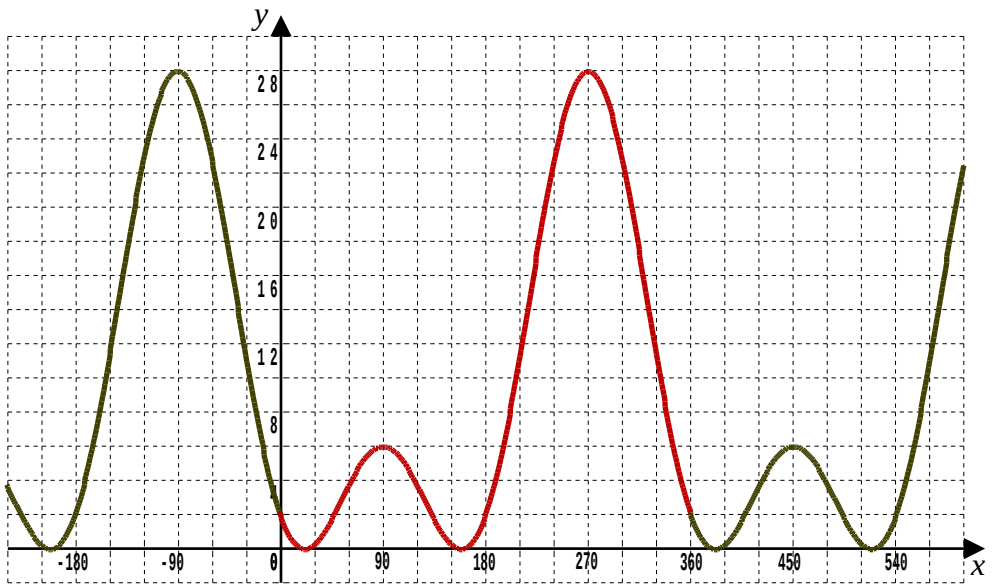
[6 marks]

Question 3

The graph is of the trigonometric equation,

$$f(x) = 15\sin^2 x - 11\sin x + 2$$

with a focus on the red piece on the interval $0° \leqslant x \leqslant 360°$

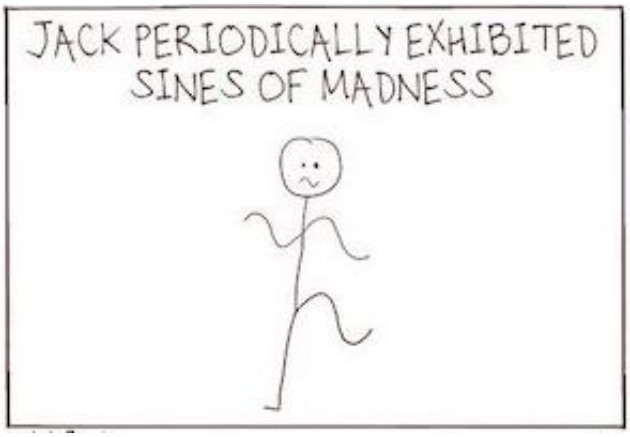


It's not clear from the graph how many, if any, solutions there are as the minimum points could be above or below the *x*-axis.

By viewing the following equation as a *quadratic in disguise* find the solutions, if there are any, to the equation

$$15\sin^2 x - 11\sin x + 2 = 0, \qquad 0° \leqslant x \leqslant 360°$$

[10 marks]



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