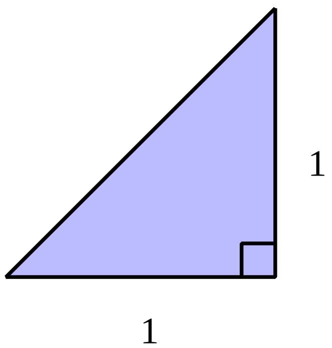
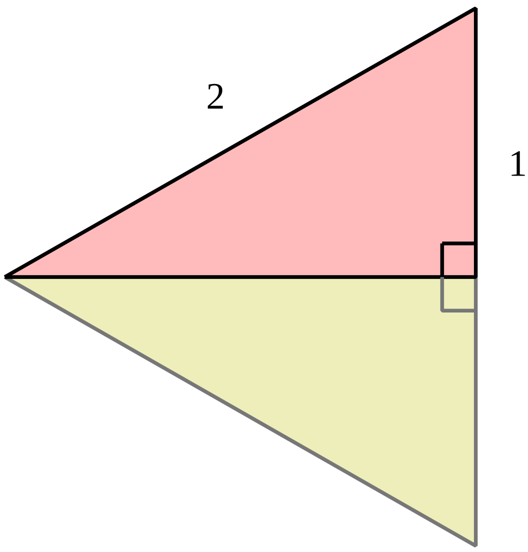
Lesson 3

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

3.1 Exactitude

The following two simple right angled triangles are of great utility, for they can be used to determine exact values for the *sin*, *cos* and *tan* of certain angles.





The left-hand triangle has a base of length exactly 1 unit and a height of exactly 1 unit. On this triangle in the appropriate place write the exact length of the hypotenuse and the exact size of the two non right angled angles.

The right-hand triangle is equilateral with sides of length exactly 2 units, but has been cut in half. The focus is upon the upper half which has a hypotenuse of exact length 2 units and height of exactly 1 unit. On this triangle in the appropriate place write the exact length of the base and the exact size of the two non right angled angles.

Watch the teaching video and complete this “Exact Values” table without a calculator,

x	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin x$					
$\cos x$					
$\tan x$					

Teaching Video : http://www.NumberWonder.co.uk/v9044/3.mp4

[5 marks]

3.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

(i) Sketch the graph of $y = \sin x$ over the interval $0° \leqslant x \leqslant 360°$

[2 marks]

(ii) Use the symmetry of your part (i) sketch, and the “Exact Values” table from the Teaching Video to complete this “Extended Exact Values” table for *sin x*. You may like to use a calculator to check your answers.

x	0	30	45	60	90	120	135	150
$\sin x$								

x	180	210	225	240	270	300	315	330
$\sin x$								

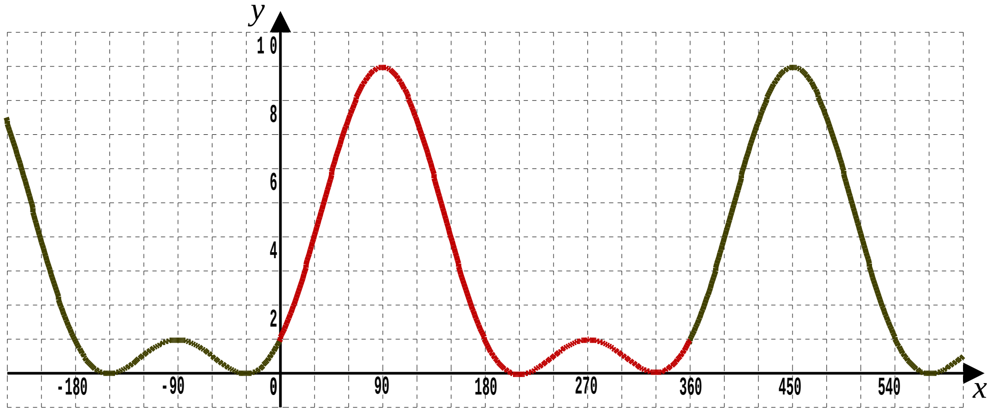
[4 marks]

Question 2

The graph is of the trigonometric equation,

$$f(x) = 4\sin^2 x + 4\sin x + 1$$

with a focus on the red piece on the interval $0° \leqslant x \leqslant 360°$



Use the mathematics of a *quadratic in disguise* to solve the equation

$$4\sin^2 x + 4\sin x + 1 = 0, \qquad 0° \leqslant x \leqslant 360°$$

Check that your answers accord with what is suggested by the above graph.

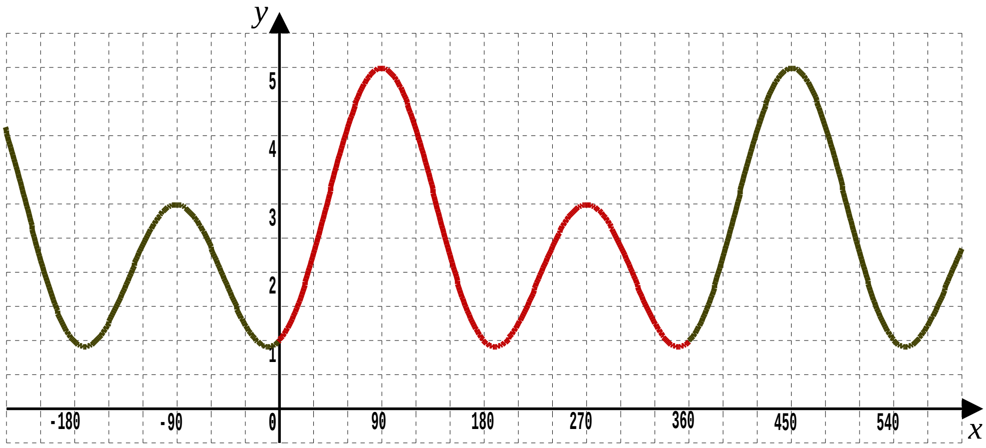
[6 marks]

Question 3

The graph below is of the function

$$f(x) = 3\sin^2 x + \sin x + 1$$

with a focus, marked in red, on the interval $0° \leqslant x \leqslant 360°$



(i) Explain how, from looking at the graph, it can be seen that it would be futile to try and solve the equation,

$$3\sin^2 x + \sin x + 1 = 0 \qquad \text{for } 0° \leqslant x \leqslant 360°$$

[2 marks]

(ii) Is trying to solve the equation

$$3\sin^2 x + \sin x + 1 = 0 \qquad \text{for } -180° \leqslant x \leqslant 540°$$

a more worthwhile endeavour ?

Explain your answer.

[2 marks]

(iii) On the graph draw the horizontal straight line $y = 5$

By looking at the graph, now with your added line on, write down a probable solutions to the equation,

$$3\sin^2 x + \sin x + 1 = 5 \qquad \text{for } 0° \leqslant x \leqslant 360°$$

[2 marks]

(iv) Use the mathematics of a *quadratic in disguise* to solve the equation

$$3\,sin^2 x + sin\,x + 1 = 5, \qquad 0° \leqslant x \leqslant 360°$$

Begin by forming a quadratic equation that equals zero before factorising that into two brackets.

You are expecting the mathematics to give you the solution identified in part (iii) but something must happen, mathematically, that prevents any other solutions from appearing.

Good luck on this voyage of discovery !

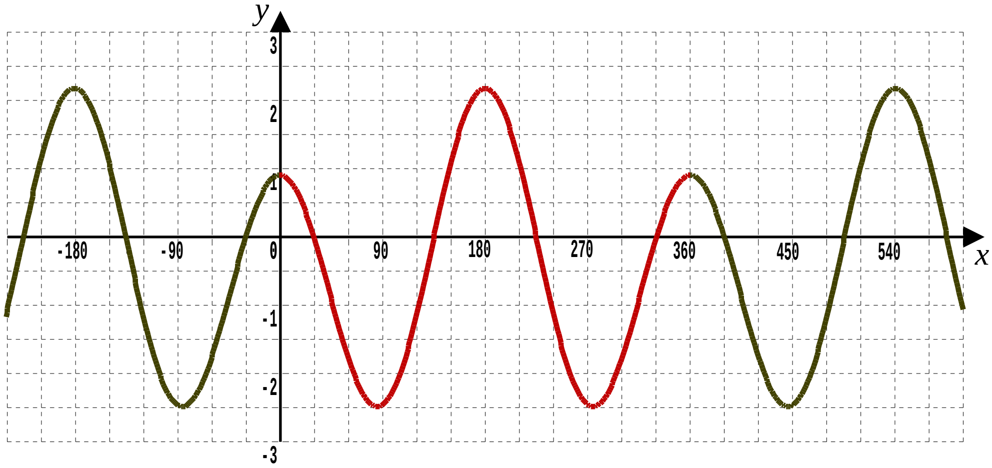
[6 marks]

Question 4

The graph is of the trigonometric equation,

$$f(x) = (2\cos x - \sqrt{3})(2\cos x + \sqrt{2})$$

with a focus on the red piece on the interval $0° \leqslant x \leqslant 360°$



Solve the equation $(2\cos x - \sqrt{3})(2\cos x + \sqrt{2}) = 0$ for $0° \leqslant x \leqslant 360°$

Check that your answers accord with what is suggested by the above graph.

[6 marks]