## 4.1 An Identity

When a trigonometric equation is solved, such as,

$$15\,sin^2 x \;-\; 11\,sin\,x \;+\; 2 \;=\; 0, \qquad 0° \leqslant x \leqslant 360°$$

the task is in finding the few values of *x* that make the equation true.

For this example, those values are, $x = 19.5°,\ 23.6°,\ 156.4°,\ 160.5°$ †

A trigonometric identity is a very different beast : It is true for all values of *x*.
The perfect example of a trigonometric identity is the following:

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**A Key Trigonometric Identity**

$$cos^2 x + sin^2 x = 1$$

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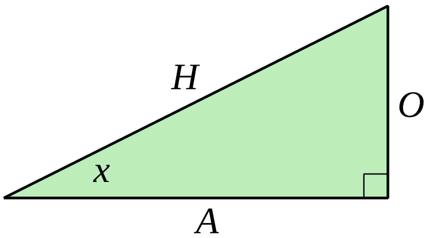
*Proof*

Consider a right angled triangle with sides labelled;

*O* for Opposite angle *x*
*A* for Adjacent to angle *x*
*H* for Hypotenuse




$$\begin{aligned}
\text{LHS} &= cos^2 x + sin^2 x \\
&= \left(\frac{A}{H}\right)^2 + \left(\frac{O}{H}\right)^2 \\
&= \frac{A^2}{H^2} + \frac{O^2}{H^2} \\
&= \frac{A^2 + O^2}{H^2} \qquad \text{By Pythagoras' Theorem, } A^2 + O^2 = H^2 \\
&= \frac{H^2}{H^2} \\
&= 1 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

† This equation was solved in Lesson 2, Exercise 2.2, Question 3

**4.2 Example**

Solve the following trigonometric equation;

$$5\,\sin x \;=\; 2\,\cos^2 x \;-\; 4 \qquad\qquad 0° \;\leqslant\; x \;\leqslant\; 360°$$

Teaching Video : http://www.NumberWonder.co.uk/v9044/4.mp4

**[ 6 marks ]**

**4.3 Exercise**

*Any solution based entirely on graphical*
*or numerical methods is not acceptable*
Marks Available : 40

**Question 1**

*A-Level Examination Question from January 2010, Paper C2, Q2 (Edexcel)*

**( a )** Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

**[ 2 marks ]**

**( b )** Solve, for $0° \leqslant x \leqslant 360°$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

**[ 4 marks ]**

**Question 2**

Find all the solutions, in the interval $0 \leqslant x \leqslant 360°$, of the equation

$$cos^2 x - 3\, cos\, x = sin^2 x + 4$$

**[ 6 marks ]**

**Question 3**

The next most useful trigonometric identity after $\cos^2 x + \sin^2 x = 1$ is;

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**Another Key Trigonometric Identity**

$$\frac{\sin x}{\cos x} = \tan x \qquad (\text{ Provided } \cos x \neq 0 )$$

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Your task now is to prove that this result is true.

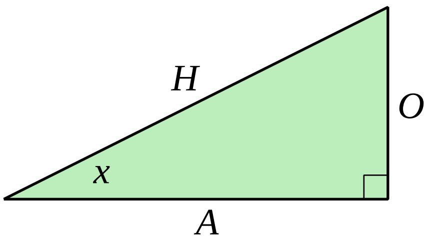
Your proof will be in a similar style to that for $\cos^2 x + \sin^2 x = 1$

To get you started here are the first few parts of the proof;

*Proof*

Consider a right angled triangle with sides labelled;

*O* for Opposite angle *x*
*A* for Adjacent to angle *x*
*H* for Hypotenuse




$$\text{LHS} = \frac{\sin x}{\cos x}$$

Complete the proof, started above.

**[ 3 marks ]**

**Question 4**

*Additional Mathematics Examination Question from June 2014, Q9 (OCR)*

**( i )** Show that $\dfrac{1 - cos^2 x}{1 - sin^2 x} = tan^2 x$

**[ 1 mark ]**

**( ii )** Hence solve the equation $\dfrac{1 - cos^2 x}{1 - sin^2 x} = 3 - 2\, tan\, x$ for values of $x$ in the range $0° \leqslant x \leqslant 180°$

**[ 4 marks ]**

**Question 5**

*A-Level Examination Question from October 2016, Paper C12, Q10 (edited) (Edexcel)*

**( a )** Given that

$$8 \tan x = -3 \cos x$$

show that

$$3 \sin^2 x - 8 \sin x - 3 = 0$$

**[ 3 marks ]**

**( b )** Hence solve, for $0 \leqslant x < 360°$,

$$8 \tan x = -3 \cos x$$

giving your answers to one decimal place, as appropriate.

**[ 5 marks ]**

**Question 6**

*A-Level Examination Question from January 2005, Paper C2, Q4 (Edexcel)*

**( a )** Show that the equation

$$5\cos^2 x = 3(1 + \sin x)$$

can be written as

$$5\sin^2 x + 3\sin x - 2 = 0$$

**[ 2 marks ]**

**( b )** Hence solve, for $0° \leqslant x \leqslant 360°$, the equation

$$5\cos^2 x = 3(1 + \sin x)$$

giving your answers to 1 decimal place where appropriate.

**[ 5 marks ]**

**Question 7**

*A-Level Examination Question from January 2011, Paper C2, Q7 (Edexcel)*

**( a )** Show that the equation

$$3\,sin^2 x + 7\,sin\,x = cos^2 x - 4$$

can be written in the form

$$4\,sin^2 x + 7\,sin\,x + 3 = 0$$

**[ 2 marks ]**

**( b )** Hence solve, for $0° \leqslant x \leqslant 360°$

$$3\,sin^2 x + 7\,sin\,x = cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

**[ 5 marks ]**