Lesson 5

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

5.1 Solving Quadratic Equations

The solution methods for trying to solve quadratic equations include,

- Factorising into two pairs of brackets
- Completing the square
- Using the Q Formula

The Q Formula

A quadratic equation that is written in the form

$$a x^2 + bx + c = 0 \qquad \text{where } a, b \text{ and } c \text{ are constants}$$

has real solutions, if any exist, given by the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the Q Formula, the expression under the square root sign, the $b^2 - 4ac$ piece, is called the discriminant, D, as it determines how many real solutions there are.

- If $b^2 - 4ac > 0$ then there are two distinct real solutions
- If $b^2 - 4ac = 0$ then there is one (repeated) real solution
- If $b^2 - 4ac < 0$ then there are no real solutions

Given that some trigonometric equations can be viewed as *quadratics in disguise* it should come as no surprise that the underlying quadratic may, in some questions, not factorise into two guessable brackets; instead, the Q formula may be needed.

Try the following example, then check your answer with mine, over the page;

5.2 Example For You To Try

For the equation $x^2 + x - 1 = 0$

(i) What is the value of the discriminant, D ?

[2 marks]

(ii) How many solutions will the equation have ?

[1 mark]

5.3 Answer to 5.2 Example

For the equation $x^2 + x - 1 = 0$

(i) What is the value of the discriminant, D ?

$a = 1,\ b = 1,\ c = -1$ so the discriminant, D, will be;

$$D = b^2 - 4ac$$
$$= 1^2 - 4 \times 1 \times (-1)$$
$$= 1 + 4$$
$$= 5$$

[2 marks]

(ii) How many solutions will the equation have ?

As $D > 0$, the equation $x^2 + x - 1 = 0$ will have 2 distinct solutions

Notice that you were not asked to solve the equation !

[1 mark]

5.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

For the equation $3x^2 + 4x + 2 = 0$

(i) What is the value of the discriminant, D ?

[2 marks]

(ii) How many solutions will the equation have ?

[1 mark]

Question 2

By considering the discriminant, D, of the underlying quadratic equation, prove that the following trigonometric equation has no solutions;

$$2\tan^2 x - 5\tan x + 4 = 0$$

[3 marks]

Question 3

(i) Use a trigonometric identity to turn the following equation into one suitable for analysing as a *quadratic in disguise*;

$$\sin^2 x + 3\cos x - 8 = 0$$

[3 marks]

(ii) By considering the discriminant, D, of your part (i) equation show that $\sin^2 x + 3\cos x - 8 = 0$ has no solutions.

[3 marks]

Question 4

A quadratic equation of the form $a x^2 + bx + c = 0$

can be solved by using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(i) Show how to use this formula to find the ***exact*** solutions to the equation

$$3x^2 - 4\sqrt{3}x + 3 = 0$$

giving your answers in the form $k\sqrt{3}$ for rational values of k

[5 marks]

(ii) Hence, or otherwise, solve over the interval $0° \leqslant x \leqslant 360°$ the equation

$$3\tan^2 x - 4\sqrt{3}\tan x + 3 = 0$$

[4 marks]

Question 5

(i) Show how to use the Q Formula to find the solutions to the equation

$$7x^2 + 5\sqrt{7}x + 6 = 0$$

giving your answers in the form $k\sqrt{7}$ for rational values of k.

[5 marks]

(ii) Hence, or otherwise, solve over the interval $0° \leqslant x \leqslant 360°$ the equation

$$7\sin^2 x + 5\sqrt{7}\sin x + 6 = 0$$

[4 marks]