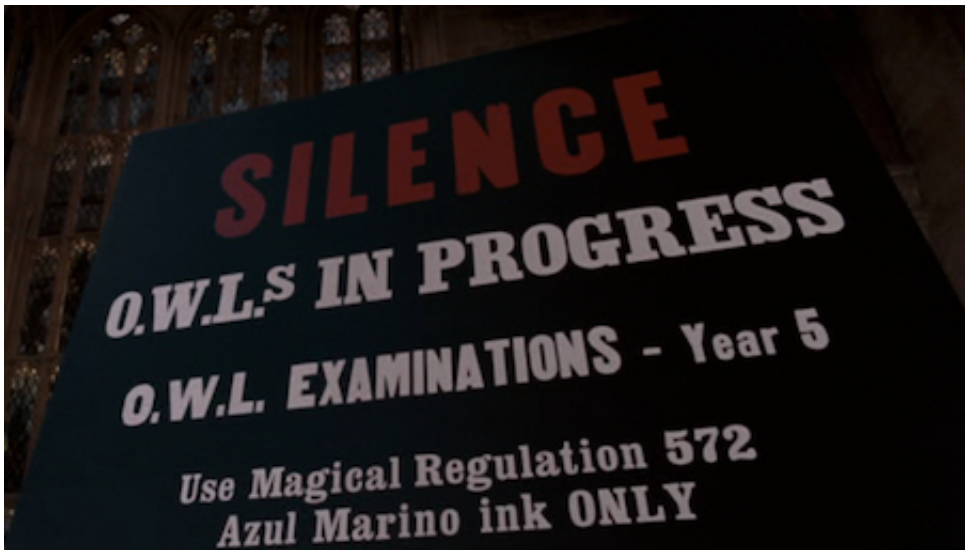
Lesson 7

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

7.1 From The Examination Hall



Here's some wand waving wizard mathemagic that mysteriously proves 2 = 1

The Classic Proof that 2 = 1

Given any number, x, let y be the same number.

0)	In other words	$x = y$
1)	Multiply both sides by x	$x^2 = xy$
2)	Subtract y^2 from both sides	$x^2 - y^2 = xy - y^2$
3)	Factorise both sides	$(x + y)(x - y) = y(x - y)$
4)	Divide both sides by $(x - y)$	$\frac{(x + y)(x - y)}{(x - y)} = \frac{y(x - y)}{(x - y)}$
5)	Simplify	$(x + y) = y$
6)	Recall that $x = y$	$(y + y) = y$
7)	Simplify	$2y = y$
8)	Divide both sides by y	$\frac{2y}{y} = \frac{y}{y}$
9)	Simplify	$2 = 1$ □

Clearly, this proof cannot be correct. On which line is there a mistake ?
The lines are numbered from 1 to 9.
Write down the line number containing a mistake :

7.2 An Examination Temptation

An old examination question that tried to tempt candidates into making the same mistake as in the false proof is this;

Solve the equation $\tan^2 x = \tan x$ for $0° \leqslant x < 360°$

Teaching Video: http://www.NumberWonder.co.uk/v9044/7.mp4

[5 marks]

7.3 The Equation $a \sin x = b \cos x$

In spite of the worry over dividing by zero, in practice it's often not the primary focus in examination questions.
In particular look out for the following popular question, easily tackled by dividing both sides by $\cos \theta$

Example : Solve $\sin x = \sqrt{3} \cos x$ for $0 \leqslant x \leqslant 360°$

[4 marks]

Solution :

$$\sin x = \sqrt{3} \cos x$$

Divide both sides by $\cos x$

(OK as $\cos x = 0$ is not a solution, i.e. $x = 90°$ is not a solution)

$$\frac{\sin x}{\cos x} = \sqrt{3}$$

$$\tan x = \sqrt{3}$$

$$x = 60°,\ 240° \qquad \text{(From a sketch graph of } y = \tan x \text{)}$$

[4 marks]

7.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 43

Question 1

Additional Mathematics Examination Question from June 2016, Q3 (OCR)

Find all the values of x in the domain $0 < x < 360°$ that satisfy,

$$3 \sin x = 4 \cos x$$

[4 marks]

Question 2

Additional Mathematics Examination Question from June 2007, Q4 (OCR)

Find all the values of x in the range $0° < x < 360°$ that satisfy

$$\sin x = -4 \cos x$$

[5 marks]

Question 3

Additional Mathematics Examination Question from June 2011, Q4 (OCR)

Solve the equation

$$5\sin 2x = 2\cos 2x$$

in the interval $0° < x < 360°$

Give your answers correct to 1 decimal place.

[5 marks]

Question 4

A-Level Examination Question from January 2018, Paper C12, Q5(i) edited (Edexcel)

Solve, for $0 < x < 90°$

$$5\sin(3x) - 7\cos(3x) = 0$$

Give each solution to 3 significant figures

[5 marks]

Question 5

A-Level Examination Question from January 2009, Paper C2, Q8 (Edexcel)

(a) Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0$$

[2 marks]

(b) Hence solve, for $0° < x < 720°$

$$4\sin^2 x + 9\cos x - 6 = 0$$

giving your answers to 1 decimal place.

[6 marks]

Question 6

Solve $\cos^2 x = \cos x$ for $0° \leqslant x < 360°$

[5 marks]

Question 7

A-Level Examination Question from January 2006, Paper C2, Q8b edited (Edexcel)

Find all the values of x, to 1 decimal place, in the interval $0° < x < 360°$ for which

$$\tan^2 x = 4$$

[5 marks]

Question 8

$$3\tan^4 x - 10\tan^2 x + 3 = 0 \qquad 0° \leqslant x \leqslant 360°$$

[6 marks]