5.1 Local Minimum & Local Maximum

Differentiation is used to find the optimal solutions to problems.
On a graph, such 'best' solutions are often found where there is either a **local maximum** or a **local minimum**.

Mathematically, to find all turning points on a curve:

STEP 1 : Differentiate the **POINTS** equation to get the **GRADIENT** equation.

STEP 2 : Set the **GRADIENT** equation equal to zero and solve.

STEP 3 : Put the solution(s) from STEP 2 into the **POINTS** equation to get a list of possible turning points.
Be aware that there may be points of inflection in this list.

STEP 4 : Differentiate the **GRADIENT** equation to get the **BEND** equation.

STEP 5 : Put the solution(s) from STEP 2 back into the **BEND** equation.

- A turning point with positive bend is a local MINimum.
- A turning point with negative bend is a local MAXimum.

Alas, a bend of zero does not determine the nature of the turning point; it could be a minimum, a maximum or a point of inflection.

5.2 Example

(i) The curve $y = f(x)$ where $f(x) = x^3 + 12x$ has no turning points.
Show that this is the case by trying to find them via the mathematical method.

[3 marks]

(ii) Circle which one of the following best describes $f(x)$;

$f(x)$ is a decreasing function $\quad$ $f(x)$ is a strictly decreasing function

$f(x)$ is an increasing function $\quad$ $f(x)$ is a strictly increasing function

[1 mark]

5.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 52

Question 1

A curve has equation $y = x^3 + 3x^2 - 24x$

Using calculus, find the coordinates of the turning points of the curve and also determine their nature.

[5 marks]

Question 2

The curve with equation $y = 8x^2 + \frac{2}{x}$ has one turning point.

Find the coordinates of this turning point and determine its nature.
Use calculus, and show your working clearly.

[5 marks]

Question 3

Consider the polynomial curve, $y = 5x^5 - 7x^3$

(i) Write down the **gradient equation** of the polynomial curve.

$$\frac{dy}{dx} =$$

[2 marks]

(ii) Write down the **bend detector equation** of the polynomial curve.

$$\frac{d^2y}{dx^2} =$$

[1 mark]

(iii) Use the appropriate equation to find the **point** on the curve when $x = 1$

[1 mark]

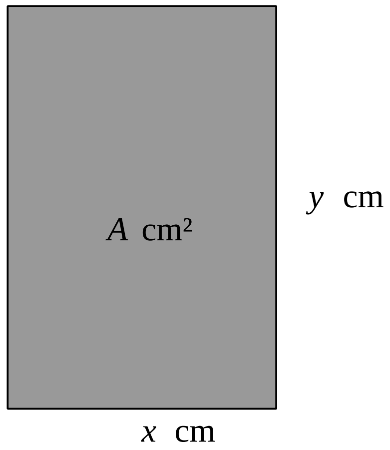
(iv) Use the appropriate equation to find the **gradient** of the curve when $x = 1$

[1 mark]

(v) Use the appropriate equation to determine, when $x = 1$, if the curve is **bending** clockwise or anticlockwise.

[2 marks]

Question 4



The diagram shows a rectangular photo frame of area A cm^2.
The width of the photo frame is x cm.
The height of the photo frame is y cm.
The perimeter of the photo frame is 72 cm.

(i) Show that $A = 36x - x^2$

[3 marks]

(ii) Find $\dfrac{dA}{dx}$

[2 marks]

(iii) Find the maximum value of A.

[3 marks]

(iv) Show that $\dfrac{d^2A}{dx^2}$ detects correctly that your part (iii) answer is a maximum.

[2 marks]

Question 5

(a) For the equation $y = 5000x - 625x^2$, find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point of $y = 5000x - 625x^2$

[3 marks]

(c) **(i)** State whether this turning point is a maximum or a minimum.

(ii) Give a reason for your answer.

[2 marks]

(d) A publisher has to set the price for a new book.
The profit, £y, depends on the price of the book, £x, where

$$y = 5000x - 625x^2$$

(i) What price would you advise the publisher to set for the book ?

(ii) Give a reason for your answer.

[2 marks]

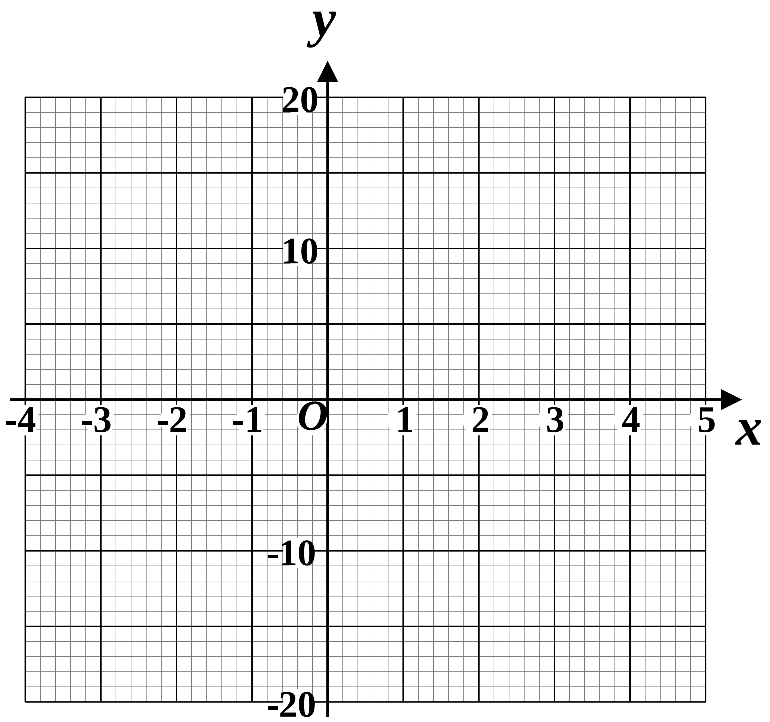
Question 6

(a) Complete the table of values for $y = x^3 - 12x + 2$

x	-3	-2	-1	0	1	2	3	4
y	11						-7	18

[2 marks]

(b) On the grid, draw the graph of $y = x^3 - 12x + 2$



[2 marks]

(c) For the curve with equation $y = x^3 - 12x + 2$

(i) Find $\dfrac{dy}{dx}$

(ii) find the gradient of the curve at the point where $x = 5$

[4 marks]

Question 7

GCSE Examination question from June 2011, 3H, Q21.

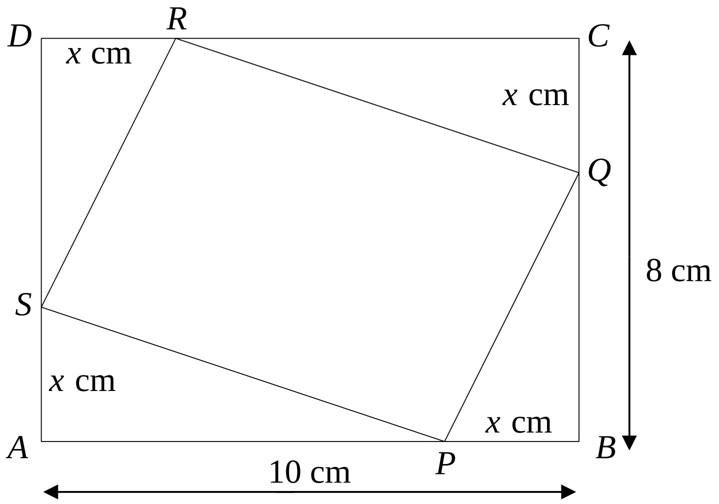


Diagram NOT accurately drawn

ABCD is a rectangle

$AB = 10$ cm

$BC = 8$ cm

P, *Q*, *R* and *S* are points on the sides of the rectangle.

$BP = CQ = DR = AS = x$ cm

(a) Show that the area, A cm^2, of the quadrilateral *PQRS* is given by the formula

$$A = 2x^2 - 18x + 80$$

[3 marks]

(b) For $A = 2x^2 - 18x + 80$

(i) find $\dfrac{dA}{dx}$

(ii) find the value of x for which A is a minimum.

(iii) Explain how you know that A is a minimum for this value of x.

[5 marks]