6.1 Stationary Points

Previously, the definition of a turning point was considered;

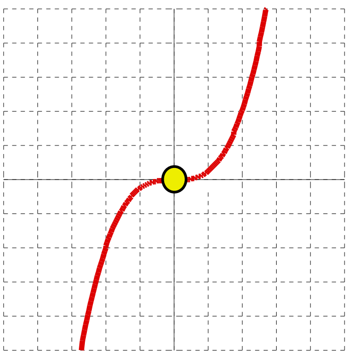
Definition: Turning Point
A turning point is a point where the gradient is zero and the sign of the gradient is different either side of the turning point.
They are so called because the gradient turns (BENDS) through zero.

Mathematicians also talk of stationary points; there is a subtle difference !

Definition: Stationary Point
A stationary point is a point where the gradient is zero.

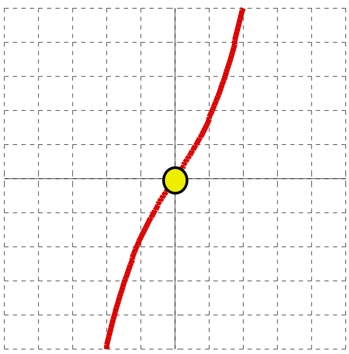
So, a local maximum or a local minimum are both turning and stationary points.
A point of inflection, which has gradient zero, is not a turning point but it is a stationary point. Technically, it's a ***stationary point of inflection***.

Note that it is possible to have a point of inflection which does not have gradient zero and so is neither a turning point nor a stationary point.
Technically, this is a ***non-stationary point of inflection***.



$y = x^3$

(0, 0) is not a turning point
but is a stationary point.
It's a stationary point of inflection.
This is an increasing function were
a gradient of zero is allowed.



$y = x^3 + x$

(0, 0) is not a turning point
and is not a stationary point.
It's a non-stationary point of inflection
This is a strictly increasing function.
The strictly emphasises that there
was no gradient of zero.

6.3 Example

$$f(x) = \frac{x^5}{5} - 2x^3 + 13x$$

Prove that $f(x)$ is a strictly increasing function.

[3 marks]

6.4 Exercise (Revision)

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 34

Question 1

$$g(x) = x^4 + 32x + 50$$

Use calculus to find the stationary point on the graph of g and determine if it is a local maximum, a local minimum or a point of inflection.

[3 marks]

Question 2

A function has equation,

$$y = 4x^5 + 8\sqrt{x} \qquad x \geqslant 0$$

Find the gradient equation of this curve.

[3 marks]

Question 3

A curve has equation

$$y = \frac{8}{x} - x + 3x^2 \qquad x > 0$$

Find the equation of the tangent to the curve at the point where $x = 2$

[4 marks]

Question 4

(i) Expand the brackets;

$$(x + h)^2$$

[1 mark]

(ii) A derivative of a smooth and continuous function $f(x)$ is defined to be;

$$f'(x) = \underset{h \to 0}{limit} \frac{f(x + h) - f(x)}{h}$$

Prove, from first principles, that the derivative of $5x^2$ is $10x$

[4 marks]

Question 5

A farmer has 80 m of fencing, and wishes to make a rectangular enclosure using a long straight wall as one of the four sides.
Let x be the length of wall used as a side, and y be the other dimension.

(i) Show that the area enclosed, A, is given by the formula

$$A = y(80 - 2y)$$

[2 marks]

(ii) Given that the farmer would like the area enclosed by this length of fence to be as large as possible, find the dimensions of the enclosure and its area.

[3 marks]

Question 6

Differentiate with respect to x

$$y = \frac{2x^3 + 5x}{x^3}$$

[3 marks]

Question 7

The curve C has equation

$$y = 4x + 3x^{\frac{3}{2}} - 2x^2 \qquad x > 0$$

(i) Find an expression for $\dfrac{dy}{dx}$

[3 marks]

(ii) Show that the point $P(4, 8)$ lies on C

[2 marks]

(iii) Show than an equation of the normal to C at point P is $3y = x + 20$

[3 marks]

The normal to C at P cuts the x-axis at point Q

(iv) Find the length PQ, giving your answer in simplified surd form

[3 marks]