A-Level

~ Year 2 ~

Pure Mathematics

ARITHMETIC
PROGRESSIONS

Carl Friedrich Gauss [1777-1855] and the problem of
summing the first one hundred positive integers.
This calculation is an example of summing an Arithmetic Progression.

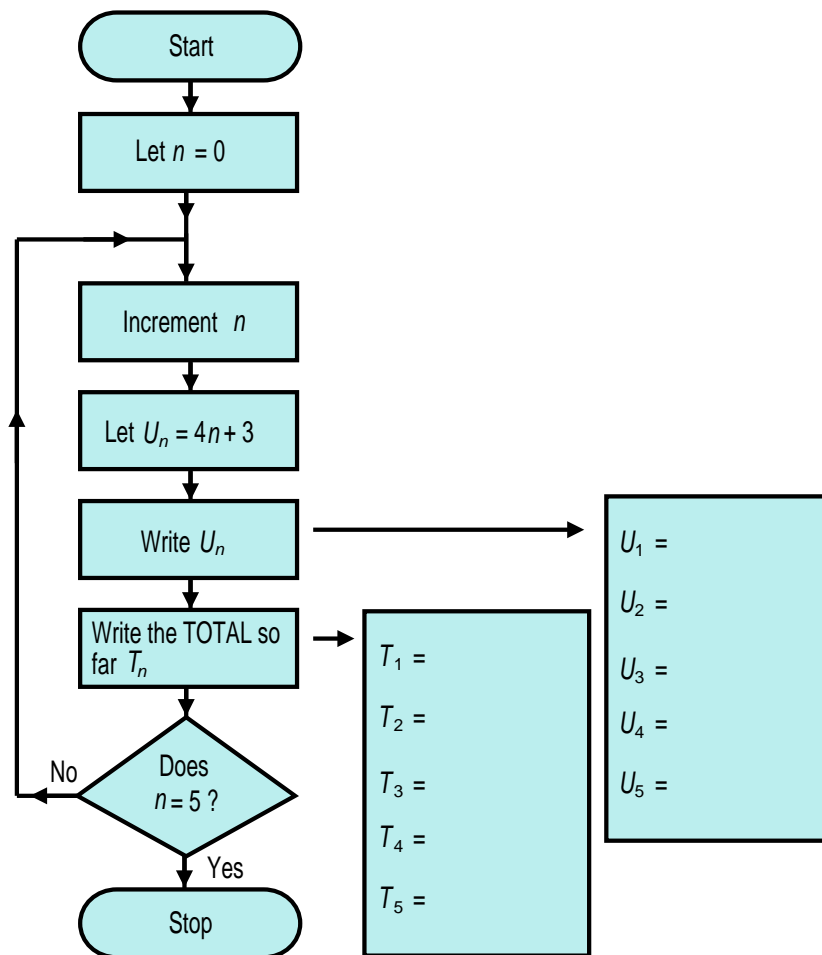# Arithmetic Progressions

Lesson 1

1.1 Sigma Notation

Teaching Video : http://www.NumberWonder.co.uk/Video/v9049(1).mp4

(**i**)   Study and complete the flow chart below;

(**ii**)   Determine the value of;
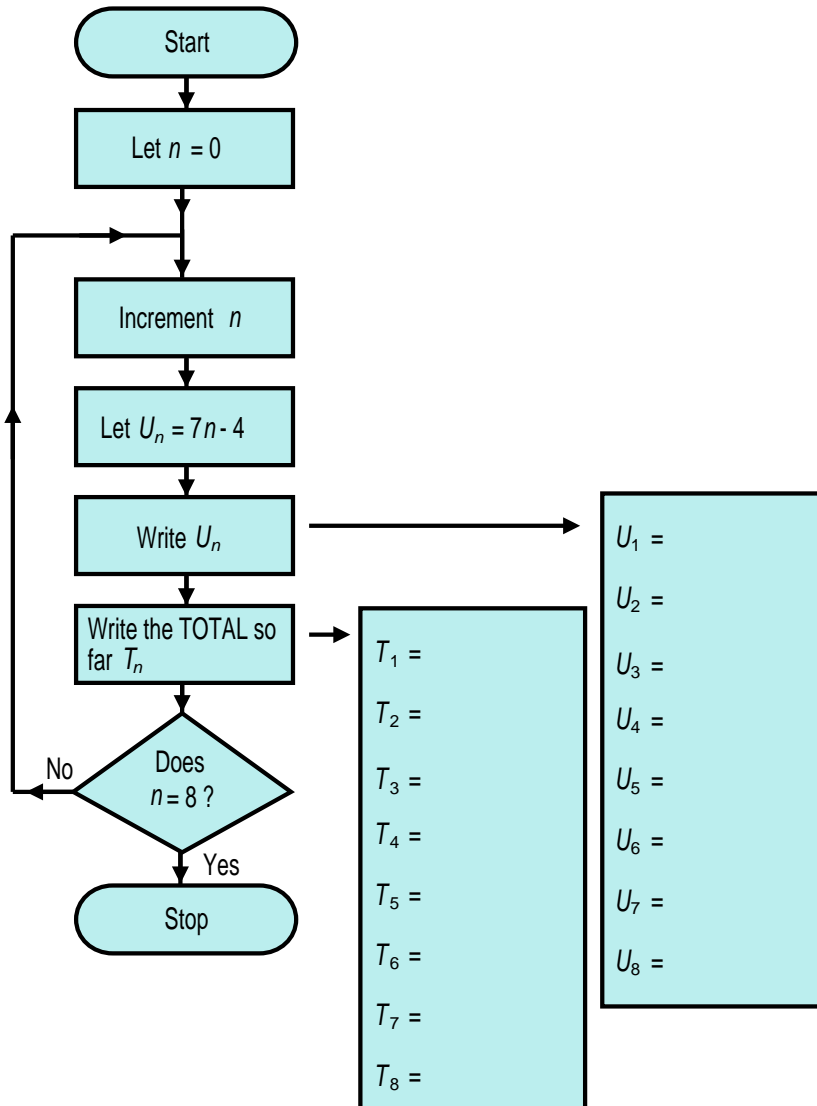
(**a**) $\displaystyle\sum_{1}^{5} \ (\ 4n \ + \ 3 \ )$ (**b**) $\displaystyle\sum_{2}^{5} \ (\ 4n \ + \ 3 \ )$

1.2 Exercise

Question 1

(i)   Study and complete the flow chart below;

(ii)   Determine the value of;

(a)

$$\sum_{1}^{8} \left(7n - 4 \right)$$

(b)

$$\sum_{4}^{8} \left(7n - 4 \right)$$

Question 2

(i)   Study and complete the flow chart below;

Start

Let $n = 0$

Increment n

Let $U_n = 20 - 3n$

Write U_n

Write the TOTAL so far T_n

No

Does $n = 6$?

Yes

Stop

$T_1 =$

$T_2 =$

$T_3 =$

$T_4 =$

$T_5 =$

$T_6 =$

$U_1 =$

$U_2 =$

$U_3 =$

$U_4 =$

$U_5 =$

$U_6 =$
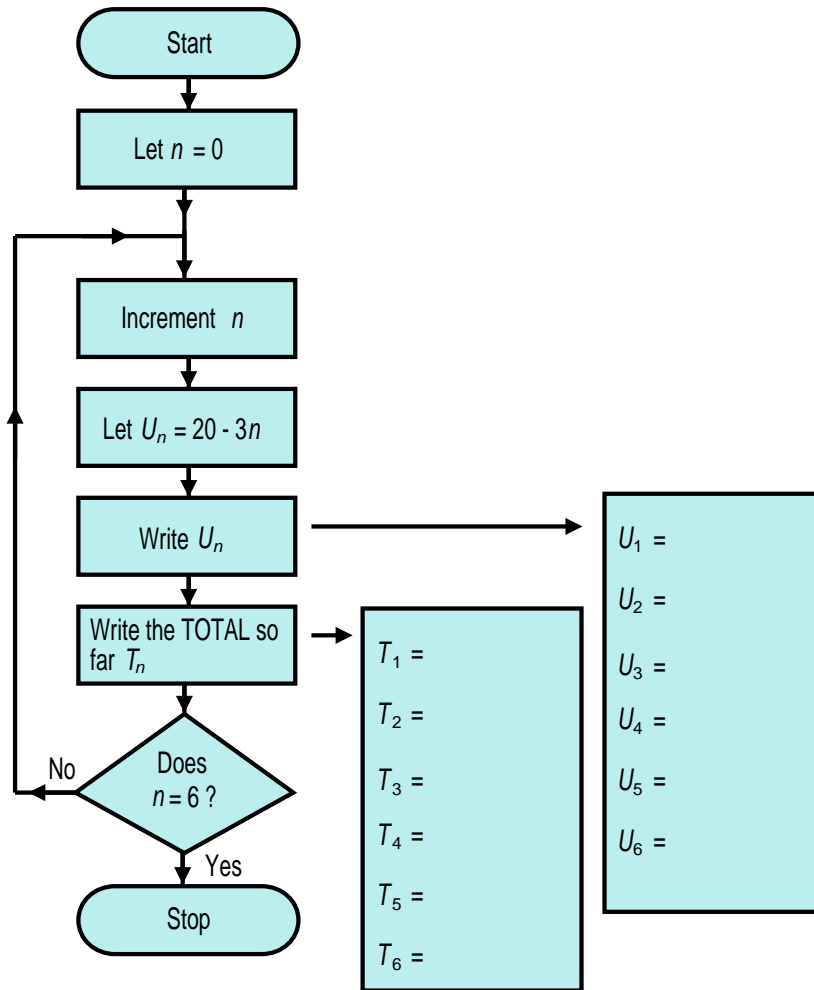
(ii)   Determine the value of;

(a)

$$\sum_{1}^{6} (20 - 3n)$$

(b)

$$\sum_{2}^{5} (20 - 3n)$$

Question 3
Determine the value of

$$\sum_{1}^{5} (10n + 3)$$

Question 4
(**i**)   Write down the formula for the nth term of the following sequence;

$$3,\ 5,\ 7,\ 9,\ 11,\ ...$$

(**ii**)   Express the following series in sigma notation;

$$3 + 5 + 7 + 9 + 11$$

Question 5
Showing some working, determine the value of

$$\sum_{10}^{15} (100 - 5n)$$

Question 6
(**i**)   Write down the formula for the nth term of the following sequence;

$$5,\ 8,\ 11,\ 14,\ 17,\ 20,\ ...$$

(**ii**)   Write the following series in sigma notation;

$$5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29$$

Question 7
Write the following series in sigma notation;

$$10 + 21 + 32 + 43 + 54 + 65 + 76 + 87 + 98 + 109 + 120$$

Question 8
Write the following series in sigma notation;

$$1 - 2 - 5 - 8 - 11 - 14 - 17$$

Question 9
The sequences and series considered have all been in in Arithmetic Progression.
Two numbers determine an Arithmetic Progression.
• The first term, a
• The common difference, d

Example
When $a = 6$ and $d = 4$ the sequence specified is;

$$6, \ (6 + 4), \ (6 + 4 + 4), \ (6 + 4 + 4 + 4 + 4), \ ...$$

$$6, \ 10, \ 14, \ 22, \ ...$$

Write down the first five terms of the Arithmetic Progression specified by,
(i) $a = 5, \ d = 6$

(ii) $a = 16, \ d = -3$

(iii) $a = 11, \ d = -7$

Question 10

Write down the formula for the nth term of the following Arithmetic Progressions;

(i) $a = 3, \ d = 8$

(ii) $a = 7, \ d = 5$

(iii) $a = 9, \ d = -4$

Question 11

(i)   Explain why the following series has 4 terms;

$$\sum_{3}^{6} (6n + 1)$$

(ii)   How many terms has the following series ?

$$\sum_{5}^{11} (6n - 1)$$

(iii)   How many terms has the following series ?

$$\sum_{15}^{47} (17n + 13)$$