Lesson 3

A-Level Pure Mathematics
Sequences & Series : Year 2

3.1 APs : Formulae Summary

Algebraically, an Arithmetic Progression is a number sequence of the form;

$$a, \quad a + d, \quad a + 2d, \quad a + 3d, \quad \dots, \quad a + (n - 1)\,d$$

where a is the initial term,
d is the common difference,
and n is number of terms.

The n^{th} term, L_n, is given by,

$$L_n = a + (n - 1)\,d \qquad n \geqslant 1$$

The sum of an AP is given by,

$$S = \frac{n}{2}\{\, a + L \,\} \qquad n \geqslant 1$$

In words this can be remembered as :

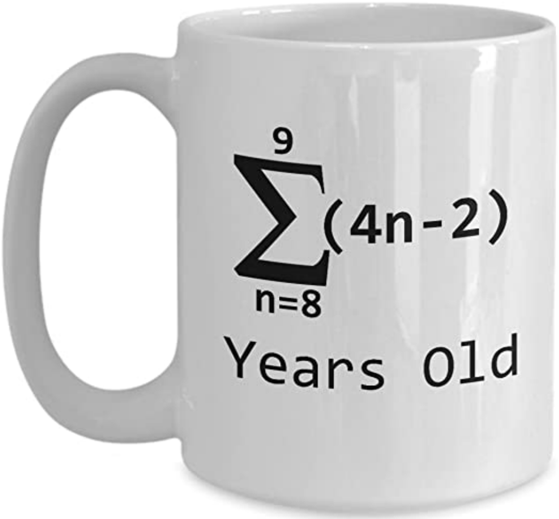
"n times the average of the first and last terms"

From substituting the first formula into the second, another formula is obtained for the sum of an AP. It is,

$$S = \frac{n}{2}\{\, 2a + (n - 1)\,d \,\}, \qquad n \geqslant 1$$

Sigma notation is often used to compact the additions.

For example, which birthday does the following mug* celebrate ?
The answer is on the next page but try to do the question yourself before looking at it.



* Available from Amazon.co.uk

3.2 Coffee Cup Sigma Notation Answer

$$\sum_{n=8}^{9}(4n - 2) = (4 \times 8 - 2) + (4 \times 9 - 2)$$

$$= 30 + 34$$

$$= 64$$

3.3 Subscript AP Questions

In this topic, the letter *n* gets used in a variety of subtly different ways.
The following examination question often puzzles newcomers to this topic.

C1 Examination question from May 2014, Q5.

A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by,

$$a_{n+1} = 5a_n - 3, \qquad n \geqslant 1$$

Given that $a_2 = 7$,

(a) find the value of a_1

[2 marks]

(b) Find the value of,

$$\sum_{r=1}^{4} a_r$$

[3 marks]

Teaching Video : http://www.NumberWonder.co.uk/Video/v9049(3a).mp4

3.4 Exercise

Question 1

C1 Examination question from January 2013, Q4

A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \qquad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4

[2 marks]

(b) Evaluate

$$\sum_{r=1}^{4} u_r$$

[3 marks]

Question 2

C1 Examination question from May 2010, Q5

A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{a_n^2 + 3}, \qquad n \geqslant 1$$

$$a_1 = 2$$

(a) Find a_2 and a_3 leaving your answers in surd form

[2 marks]

(b) Show that $a_5 = 4$

[2 marks]

Question 3

C1 Examination question from May 2006, Q7

An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of a and the value of d.

[7 marks]

Question 4

C1 Examination question from January 2010, Q7

Jill gave money to a charity over a 20-year period, from year 1 to Year 20 inclusive. She gave £150 in year 1, £160 in Year 2, £170 in year 3, and so on, so that the amounts of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10.

[2 marks]

(b) Calculate the total amount of money she gave over the 20-year period.

[3 marks]

Kevin also gave money to the charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.
The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A

[4 marks]

Question 5

C1 Examination question from May 2006, Q4

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 3a_n - 5, \qquad n \geqslant 1$$

(a) Find the value of a_2 and the value of a_3

[2 marks]

(b) Calculate the value of

$$\sum_{r=1}^{5} a_r$$

[3 marks]

Question 6

C1 Examination question from May 2014, Q8

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

[2 marks]

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

[3 marks]

In the year 2000, the selling price of each computer was £900. the selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year.
By forming and solving an equation, find the year in which this occurred.

[4 marks]

Question 7

C1 Examination question from June 2009, Q7

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k

[1 mark]

(b) Show that

$$a_3 = 4k - 21$$

[2 marks]

Given that

$$\sum_{r=1}^{4} a_r = 43$$

(c) find the value of k

[4 marks]

Question 8

C1 Examination question from January 2008, Q11

The first term of an arithmetic sequence is 30 and the common difference is -1.5

(a) Find the value of the 25th term.

[2 marks]

The r^{th} term of the sequence is 0.

(b) Find the value of r

[2 marks]

The sum of the first n terms of the sequence is S_n

(c) Find the largest positive value of S_n

[3 marks]