4.1 Harder Examination Questions

Question 1

C1 Examination question from January 2012, Q4

A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = a\,x_n + 5, \qquad n \geqslant 1$$

where a is a constant.

(**a**) Write down an expression for x_2 in terms of a

[**1 mark**]

(**b**) Show that

$$x_3 = a^2 + 5a + 5$$

[**2 marks**]

Given that $x_3 = 41$

(**c**) find the possible values of a

[**3 marks**]

Question 2

C1 Examination question from January 2012, Q9

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1 :

Salary in Year 1 is £P

Salary increases by £$(2T)$ each year, forming an arithmetic sequence

Scheme 2 :

Salary in Year 1 is £$(P + 1800)$

Salary increases by £T each year, forming an arithmetic sequence

(a) Show that the **total** earned under salary Scheme 1 for the 10-year period is

$$£(10P + 90T)$$

[2 marks]

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of T

[4 marks]

For this value of *T*, the salary in Year 10 under Salary Scheme 2 is £29 850

(c)   Find the value of *P*

[3 marks]

Question 3

C1 Examination question from May 2013, Q4

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$

$$a_{n+1} = k(a_n + 2), \qquad n \geqslant 1$$

where k is a constant.

(a)   Write down an expression for a_2 in terms of k

[1 mark]

Given that

$$\sum_{i=1}^{3} a_i = 2$$

(b)   find the two possible values of k

[6 marks]

Question 4

C1 Examination question from January 2008, Q7

A sequence is given by:

$$x_1 = 1$$

$$x_{n+1} = x_n (p + x_n)$$

where p is a constant ($p \neq 0$).

(**a**)   Find x_2 in terms of p

[**1 mark**]

(**b**)   Show that

$$x_3 = 1 + 3p + 2p^2$$

[**2 marks**]

Given that $x_3 = 1$

(**c**)   find the value of p

[**3 marks**]

(**d**)   write down the value of x_{2008}

[**2 marks**]

Question 5

C1 Examination question from January 2013, Q7

Lewis played a game of space invaders.
He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

[2 marks]

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.
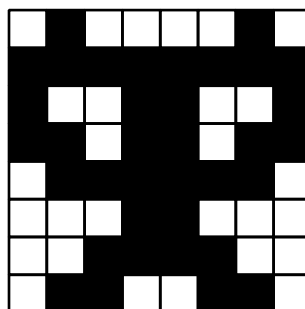
[3 marks]

Sian played an adventure game,
She scored points for each dragon that she captured.
The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c)   find the value of n

[3 marks]

Question 6

C1 Examination question from May 2007, Q8

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k$$

$$a_{n+1} = 3a_n + 5, \qquad n \geqslant 1$$

where k is a positive integer.

(**a**)   Write down an expression for a_2 in terms of k

[**1 mark**]

(**b**)   Show that $a_3 = 9k + 20$

[**2 marks**]

(**c**)   (**i**)   Find $\displaystyle\sum_{r=1}^{4} a_r$ in terms of k.

(**ii**)   Show that $\displaystyle\sum_{r=1}^{4} a_r$ is divisible by 10.

[**4 marks**]

Question 7

C1 Examination question from January 2011, Q6

An arithmetic sequence has first term a and common difference d.
The sum of the first 10 terms of the sequence is 162.

(**a**)   Show that $10a + 45d = 162$

[**2 marks**]

Given also that the sixth term of the sequence is 17

(**b**)   write down a second equation in a and d

[**1 mark**]

(**c**)   find the value of a and the value of d

[**4 marks**]

Question 8

C1 Examination question from May 2010, Q9

A farmer has a pay scheme to keep fruit pickers working throughout
the 30 day season. He pays £a for their first day, £$(a + d)$ for their second day,
£$(a + 2d)$ for their third day, and so on, thus increasing the daily payment by £d
for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(**a**) Use this information to form an equation in a and d

[**2 marks**]

A picker who works for all 30 days will earn a total of £1005

(**b**) Show that $15 (a + 40.75) = 1005$

[**2 marks**]

(**c**) Hence find the value of a and the value of d

[**4 marks**]

Question 9

C1 Examination question from May 2011, Q5

A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_1 = k$$

$$a_{n+1} = 5a_n + 3, \qquad n \geqslant 1$$

where k is a positive integer.

(**a**)   Write down an expression for a_2 in terms of k

[**1 mark**]

(**b**)   Show that $a_3 = 25k + 18$

[**2 marks**]

(**c**)   (**i**)   Find $\displaystyle\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(**ii**)   Show that $\displaystyle\sum_{r=1}^{4} a_r$ is divisible by 6.

[**4 marks**]

Question 10

C1 Examination question from May 2011, Q9

(**a**)   Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + ... + 100$$

[3 marks]

(**b**)   In the arithmetic series

$$k + 2k + 3k + ... + 100$$

k is a positive integer and *k* is a factor of 100

(**i**)   Find, in terms of *k*, an expression for the number of terms in this series

(**ii**)   Show that the sum of this series is

$$50 + \frac{5000}{k}$$

[4 marks]

(c)   Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), \ (4k + 4), \ (6k + 7), \ ......$$

giving your answer in its simplest form

[2 marks]

$$(2k + 1), \ (4k + 4), \ (6k + 7), \ ......$$