Lesson 10

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

10.1 Put It In The Bin

When a mathematical expression starts to get complicated, one of the ways to reduce the complexity is to think carefully about which parts of the expression are dominant and which, if any, are so insignificant they can be thrown away.

Suppose we have a variable x which, although unknown, is likely to be small, say, definitely positive and less than 1.

Suppose $x = 0.1$ in which case:

$$0.1^0 = 1$$
$$0.1^1 = 0.1$$
$$0.1^2 = 0.01$$
$$0.1^3 = 0.001$$
$$0.1^4 = 0.0001$$
$$0.1^5 = 0.00001$$
$$0.1^6 = 0.000001$$

The implication of this observation is that, if x is sufficiently small, large powers of x can be ignored without making much difference to an answer that is only required to a few significant figures.

Here is the mathematical way of making that observation:

The Powers Of Small *x* Rule

$$\text{If} \quad -1 < x < 1, \ \text{then} \ x^n \to 0 \ \text{ as } n \to \infty$$

10.2 Example

(i) Find the first three terms of the binomial expansion, in ascending powers of x, of the expression $\left(2 + \frac{x}{4}\right)^8$

(ii) Use your expansion to estimate the value of 2.075^8 giving your answer to the nearest integer.

Teaching Video: http://www.NumberWonder.co.uk/v9062/10.mp4

[4, 3 marks]

10.3 Exercise

Marks Available : 50

Question 1

(i) Find the first three terms of the binomial expansion, in ascending powers of x, of the expression $\left(3 + \dfrac{x}{9}\right)^{11}$

[4 marks]

(iii) Use your expansion to estimate the value of 3.01^{11} giving your answer to six significant figures.

[4 marks]

Question 2

Find the constant term (the term independent of x) in the expansion of

$$\left(\frac{x}{2} + \frac{4}{x}\right)^4$$

[4 marks]

Question 3

(i) Find the first four terms, in ascending powers of x, of the binomial expansion of $(1 - 3x)^5$

Give each term in its simplest form

[4 marks]

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1 + x)\ (1 - 3x)^5 \approx 1 - 14x$$

[4 marks]

Question 4

There are many fascinating relationships hidden in Pascal's Triangle.
This question explores one of these

(i) Work out ${}^{2}C_0 + 2\,{}^{2}C_1 + 4\,{}^{2}C_2$

[2 marks]

(ii) Work out ${}^{3}C_0 + 2\,{}^{3}C_1 + 4\,{}^{3}C_2 + 8\,{}^{3}C_3$

[2 marks]

(iii) Guess the answer, then work out

$${}^{4}C_0 + 2\,{}^{4}C_1 + 4\,{}^{4}C_2 + 8\,{}^{4}C_3 + 16\,{}^{4}C_4$$

[2 marks]

(iv) Write out the next calculation in this sequence and predict the answer ahead of working it out

[2 marks]

Question 5

Find the constant term (the term independent of x) in the expansion of

$$\left(\frac{x^2}{2} - \frac{2}{x} \right)^9$$

[5 marks]

Question 6

$$f(x) = (1 - 5x)^{30}$$

(i) Find the first four terms, in ascending powers of x, in the binomial expansion of $f(x)$

[4 marks]

(ii) Use your answer to part (i) to estimate the value of 0.995^{30} giving your answer to 6 decimal places

[3 marks]

(iii) Use your calculator to evaluate 0.995^{30} and calculate the percentage error in your answer to part (ii)

[3 marks]

Question 7

Additional Mathematics Examination Question from June 2016, Q8 (OCR)

(i) Write down the binomial expansion of $(1 + \delta)^3$

[2 marks]

(ii) Hence explain why, if δ is small, $(1 + \delta)^3 \approx 1 + 3\delta$

[$\approx$ means "is approximately equal to"]

[1 mark]

You are given that the equation $x^3 - 0.9x - 0.206 = 0$ has a root very close to $x = 1$

(iii) Substitute $x = 1 + \delta$ into the equation and use the approximation in part (ii) to find an estimate of this root, correct to 3 significant figures. Show all your working.

[4 marks]