Lesson 2

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

2.1 Making Use Of Pascal's Triangle

Pascal's Triangle, initially seeming curiosity, turns out to be extraordinarily useful. It is a hub at which many topics converge; probability and algebra initially and, later on, trigonometry and complex numbers.

For example, it is **the way** to expand the brackets of $(1 + x)^5$

Teaching Video: http://www.NumberWonder.co.uk/v9062/2.mp4

[3 marks]

2.2 Exercise

Marks Available : 50

Question 1

Here is Pascal's Triangle up to Row 9

1									
1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	28	8	1	
1	9	36	84	126	126	84	36	9	1

Using Pascal's Triangle, or otherwise, expand the brackets of $(1 + x)^8$

[3 marks]

Question 2

Pick a number in the diagonal that goes, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
Colour it blue.
Colour the six bricks around it in alternating green and red.
Multiply together the three red numbers and, separately, multiply together the three green numbers.
Now add together the blue, the red product and the green product.
The number you get is always of a certain type.
What type of number do you always get ?

For Example

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

$$3 + 6 \times 1 \times 2 + 4 \times 1 \times 3 = 27$$

A copy of Pascal's Triangle for you to do a few on...

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

[6 marks]

Question 3

Using Pascal's Triangle, or otherwise, expand the brackets of $(1 + x)^{10}$

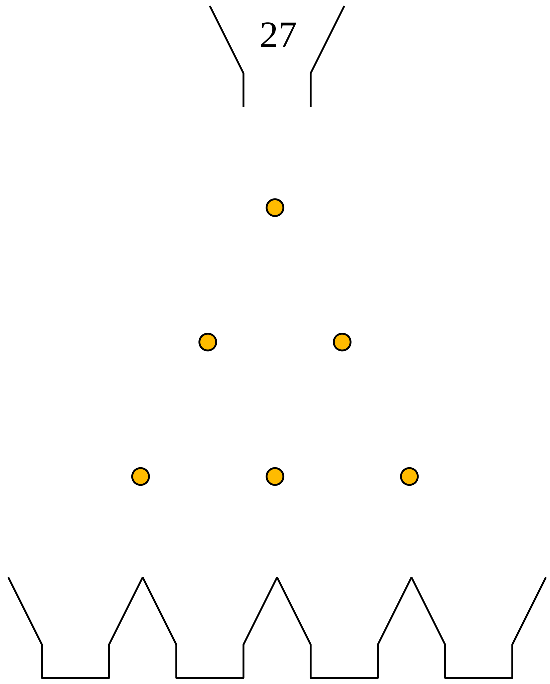
[4 marks]

Question 4

Twenty-seven steel ball bearings are released into a pinball machine.
At each pin the probability a ball will go left is half that of it going right.

(i) Above each pin write the total number of balls arriving at that pin

(ii) Show that the resulting distribution in the bottom baskets is 1 6 12 8



[4 marks]

Question 5

On the following copy of Pascal's Triangle, highlight **all** the Triangular Numbers.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

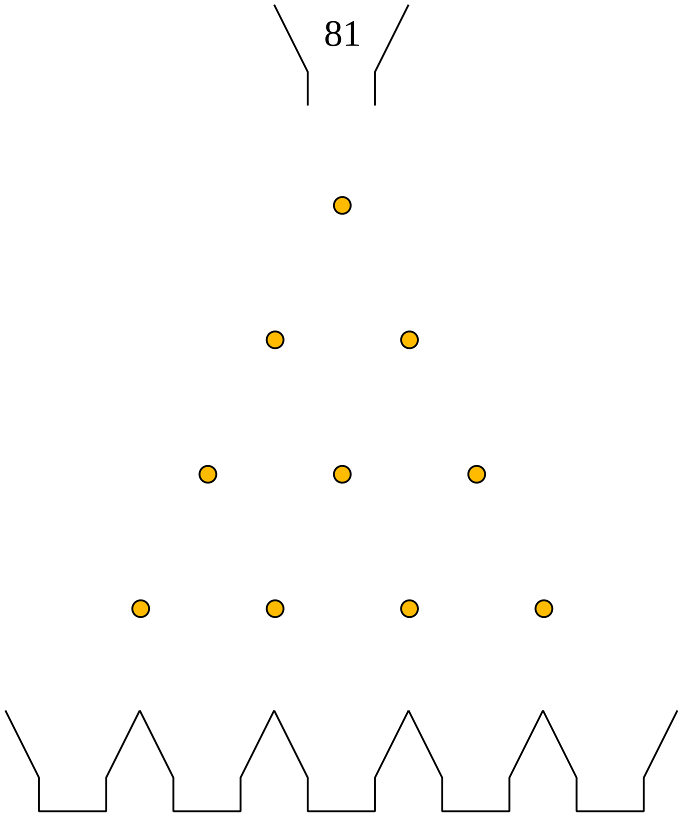
[3 marks]

Question 6

Eighty-one steel ball bearings are released into a pinball machine.
At each pin the probability a ball will go left is half that of it going right.

(i) Above each pin write the total number of balls arriving at that pin

(ii) Work out how many ball bearings end up in each of the bottom baskets.



[6 marks]

Question 7

You are given that

$$(1 - x)^0 = 1$$
$$(1 - x)^1 = 1 - 1x$$
$$(1 - x)^2 = 1 - 2x + 1x^2$$
$$(1 - x)^3 = 1 - 3x + 3x^2 - 1x^3$$
$$(1 - x)^4 = 1 - 4x + 6x^2 - 4x^3 + 1x^4$$

Using Pascal's Triangle, or otherwise, expand the brackets of $(1 - x)^7$

[3 marks]

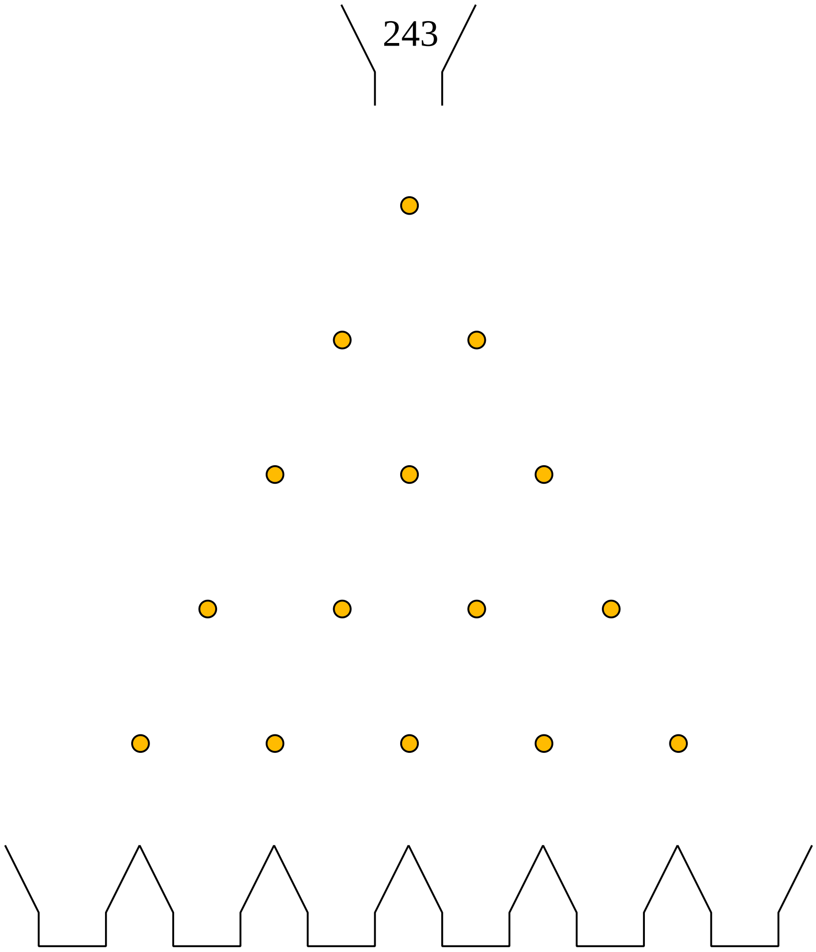
Question 8

Two Hundred and Forty-Three steel ball bearings are released into a pinball machine.

At each pin the probability a ball will go left is half that of it going right.

(i) Above each pin write the total number of balls arriving at that pin.

(ii) Work out how many ball bearings end up in each of the bottom baskets.



[11 marks]

Question 9

Use the table below to summarize your results from Q4, Q6 and Q8

Starting Number of Balls	Distribution of Balls in Baskets
27	1 6 12 8
81	
243	

[4 marks]

Teaching Interlude

The pattern summarized in the table is not so easy to spot.
There is another triangular array of numbers that results but it turns out it's not needed.
Before looking at why it's not needed observe that there is connection with expanding the brackets of $(1 + 2x)^n$ and the pinball machine ball distributions.
For example, with $n = 3$,

$$(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$$

where the 1 6 12 8 was the distribution from the pinball machine of question 4.

Clearly, as mathematicians we'd like an easy way to expand expressions such as $(1 + 2x)^3$ and the good news is that we only need Pascal's Triangle to do it.

Here is how,

- Use Pascal's Triangle for $(1 + x)^3 = 1 + 3x + 3x^2 + x^3$
- Replace the x in that with $(2x)$

Thus, $$(1 + 2x)^3 = 1 + 3(2x) + 3(2x)^2 + (2x)^3$$
$$= 1 + 6x + 12x^2 + 8x^3$$

And there is the "1 6 12 8"

Question 10

Show how to use the above method to expand $(1 + 2x)^4$
This should match the distribution from question 6

[3 marks]

Question 11

Show how to use the above method to expand $(1 + 2x)^5$
This should match the distribution from question 8

[3 marks]