Lesson 3

3.1 Pascal To The Max

Pascal's Triangle is turning out to be useful, not just in expanding the brackets of expressions of the form $(1 + x)^n$ for positive integer values of n, but also of expressions of the form $(1 + ax)^n$ where a is a constant.

For example, it is **the way** to expand the brackets of $(1 - 4x)^3$

Teaching Video: http://www.NumberWonder.co.uk/v9062/3.mp4
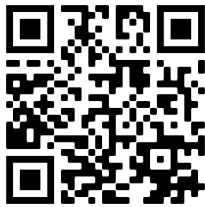
[4 marks]

3.2 Exercise

Marks Available : 40

Question 1

Expand the brackets, $(1 + 3x)^4$

[4 marks]

Question 2

Expand the brackets, $(1 - 2x)^5$

[4 marks]

Question 3

Pick a number in the diagonal that goes, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...

Colour it blue.

Colour the six bricks around it in alternating red, green, purple.
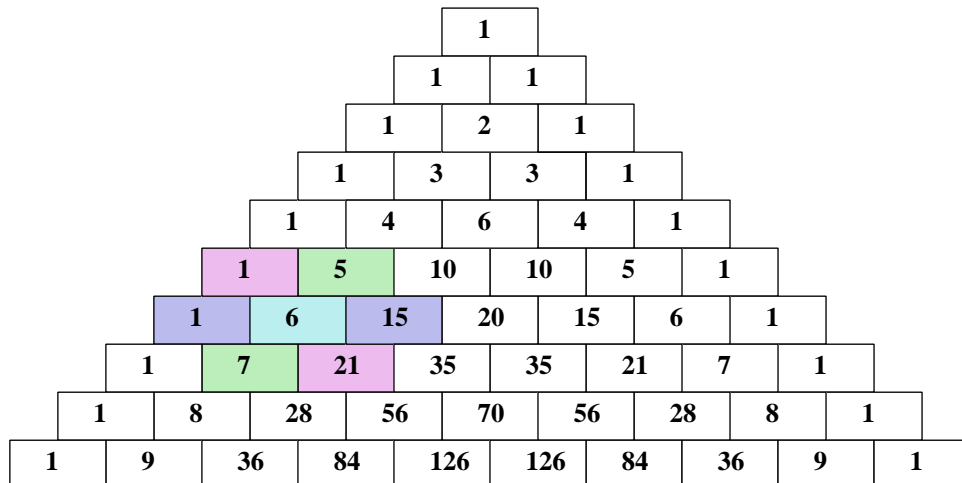
For each colour work out the difference between the two numbers of that colour

Add up these three colour differences.
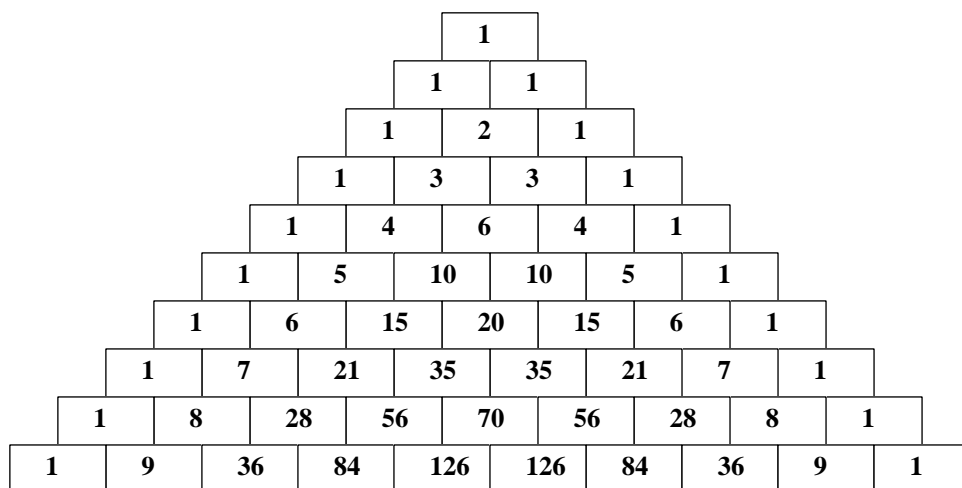
The number you get is always of a certain type.

What type of number do you always get ?

For Example

									1									
								1		1								
							1		2		1							
						1		3		3		1						
					1		4		6		4		1					
				1		5		10		10		5		1				
			1		6		15		20		15		6		1			
		1		7		21		35		35		21		7		1		
	1		8		28		56		70		56		28		8		1	
1		9		36		84		126		126		84		36		9		1

$$(21 - 1) + (7 - 5) + (15 - 1) = 36$$

A copy of Pascal's Triangle for you to do a few on...

|									1									
								1		1								
							1		2		1							
						1		3		3		1						
					1		4		6		4		1					
				1		5		10		10		5		1				
			1		6		15		20		15		6		1			
		1		7		21		35		35		21		7		1		
	1		8		28		56		70		56		28		8		1	
1		9		36		84		126		126		84		36		9		1

[4 marks]

Question 4

Twenty-five steel ball bearings are released into a pinball machine.

At each pin the probability a ball will go left is $\dfrac{2}{5}$ and go right is $\dfrac{3}{5}$

(**i**)   Above each pin write the total number of balls arriving at that pin

(**ii**)   Show that the resulting distribution in the bottom baskets is 4   12   9

25
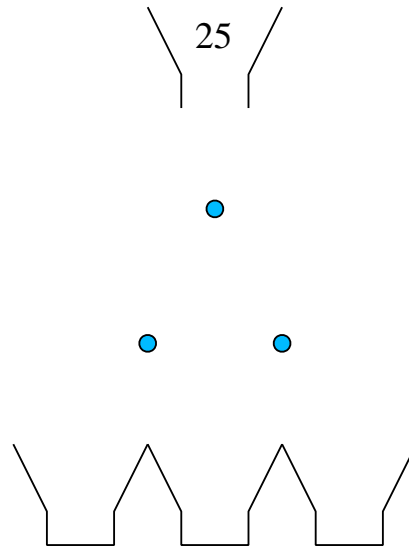
[4 marks]

Question 5

Expand $(1 + 6x)^7$ in ascending powers of x up to the term in x^3 and simplify.

Note :   • Ascending powers of x means $a + bx + c\,x^2 + d\,x^3 + e\,x^4 + ...$

   • The constants $a,\ b,\ c,\ d,\ e,\ ....$ are called the coefficients

   • This question only wants $a + bx + c\,x^2 + d\,x^3$ $\Leftarrow$ STOP !

   • Simplify means bracketed things like $(6x)^3$ in the answer will cost marks
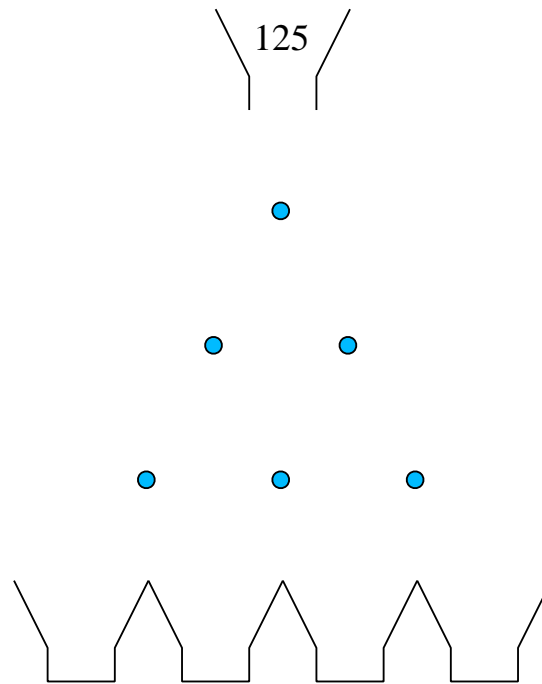
[4 marks]

Question 6

Twenty-five steel ball bearings are released into a pinball machine.

At each pin the probability a ball will go left is $\dfrac{2}{5}$ and go right is $\dfrac{3}{5}$

(**i**)　　Above each pin write the total number of balls arriving at that pin.

(**ii**)　　Work out how many ball bearings end up in each of the bottom baskets.

125

[6 marks]

Question 7

Additional Mathematics Examination Question from June 2010, Q2 (OCR)

Expand $(1 - x)^{12}$ in ascending powers of x up to the term in x^3

Simplify your answer.

[3 marks]

Question 8

A-Level Examination Question from January 2019, C12, Q5 (Edexcel)

Find the first 4 terms, in ascending powers of *x,* of the expansion of

$$\left(1 - \frac{x}{2} \right)^8$$

Give each term in its simplest form

<div align="right">[4 marks]</div>

Question 9

You are given that $(x + y)^0 = 1$

$$(x + y)^1 = 1\,x + 1\,y$$

$$(x + y)^2 = 1\,x^2 + 2\,x\,y + 1\,y^2$$

$$(x + y)^3 = 1\,x^3 + 3\,x^2\,y + 3\,x\,y^2 + 1\,y^3$$

$$(x + y)^4 = 1\,x^4 + 4\,x^3\,y + 6\,x^2\,y^2 - 4\,x\,y^3 + 1\,y^4$$

Using Pascal's Triangle, or otherwise, expand the brackets of $(x + y)^7$

<div align="right">[3 marks]</div>

Question 10

Using the result that,

$$(x + y)^3 = 1\,x^3 + 3\,x^2\,y + 3\,x\,y^2 + 1\,y^3$$

expand the brackets of,

$$(2 + 3x)^3$$

It you get this correct, and got question 6 correct, the two answers will have the same distribution of balls in baskets as coefficients of x.

If they are not the same, one or the other or both are wrong !

[4 marks]