6.1 Spinning Coins

A coin is spun three times

Each time it may spin heads, *H*, or tails, *T*

Here is a tree diagram that shows all the possible outcomes,
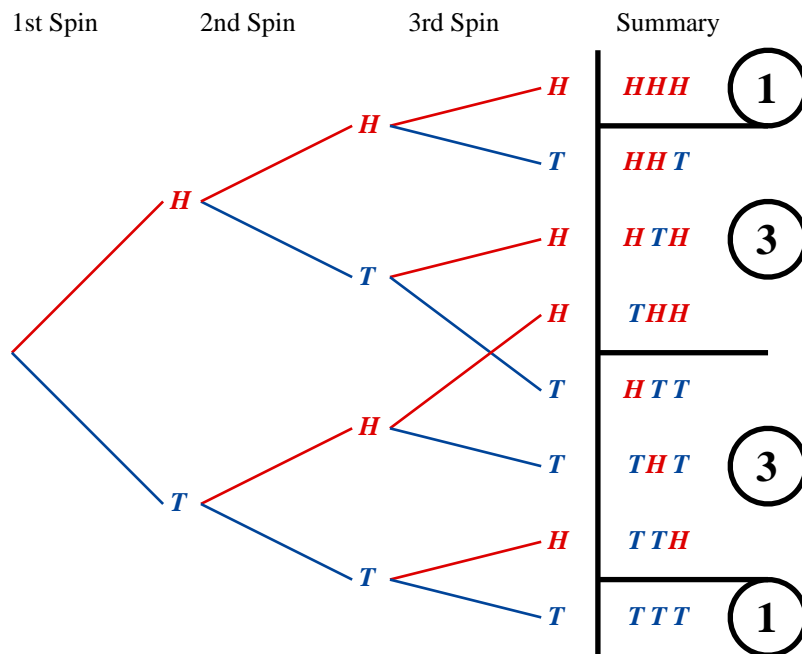
Of particular interest is the summary where there is,

- 1 way of spinning three heads and zero tails
- 3 ways of spinning two heads and one tail (if the order does not matter)
- 3 ways of spinning one head and two tails (if the order does not matter)
- 1 way of spinning zero heads and three tails

It is not a coincidence that these numbers are Row 3 of Pascal's Triangle.
The implication is that Pascal's Triangle is going to be very useful in tackling
probability questions beyond GCSE.

From the fact that $(x + y)^3 = 1\,x^3\,y^0 + 3\,x^2\,y^1 + 3\,x^1\,y^2 + 1\,x^0\,y^3$
with *x* replaced with *H* and *y* replaced with *T* we get,

$$(H + T)^3 = 1\,H^3\,T^0 + 3\,H^2\,T^1 + 3\,H^1\,T^2 + 1\,H^0\,T^3$$

This is an mathematical algebraic model for spinning a coin three times !

The term $3\,H^2\,T^1$ tells you that there are three ways of spinning two heads and
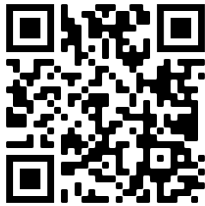one tail (if the order in which the heads and tails occur does not matter)

6.2 Example

A coin is biased such that, when it is spun, the probability of it showing heads is twice the probability of it showing tails;

In other words,
$$P(H) = \frac{2}{3}$$

$$P(T) = \frac{1}{3}$$

When spun three times what is the probability that two heads and one tail occur ?

http://www.NumberWonder.co.uk/v9062/6.mp4

[4 marks]

6.3 Exercise

Marks Available : 50

Question 1

A coin is biased such that, when it is spun, the probability of it showing heads is three times the probability of it showing tails;

In other words,
$$P(H) = \frac{3}{4}$$

$$P(T) = \frac{1}{4}$$

When spun four times what is the probability that two heads and two tails occur ?
Give your answer as an exact fraction.

[4 marks]

Question 2

A coin is biased such that, when it is spun, the probability of it showing heads is half the probability of it showing tails;

In other words, $\qquad P(H) = \dfrac{1}{3}$

$$P(T) = \dfrac{2}{3}$$

When spun six times what is the probability that four heads and two tails occur ?
Give your answer correct to three decimal places.

[4 marks]

Question 3

A coin is biased such that

$$P(heads) = 0.2$$

$$P(tails) = 0.8$$

It is spun three times.
What is the most likely number of tails to occur ?
Back up your answer with mathematical calculations.

[5 marks]

Question 4

A fair coin is spun nine times.

What is the percentage probability that exactly seven heads and two tails are spun ?

[4 marks]

Question 5

A fair coin is spun six times.

Show that the probability that more heads are spun than tails is roughly a third.

Show your working.

[5 marks]

Question 6
When a learner driver takes their practical driving test, experience suggests that
the probability of them passing is 0.6
(**i**)   What is the probability of them failing ?

[1 mark]

(**ii**)   At a test centre, ten students take their driving test in a day.
         What is the percentage probability that exactly eight pass ?

[5 marks]

Question 7
In a multiple choice test each question presents four options, only one of which is
correct. In a twelve question test what is the probability that a person, picking answers
at random, gets less than two questions wrong ?
Give your (very small) answer correct to three significant figures.

[5 marks]

Question 8
Additional Mathematics Examination Question from June 2018, Q9 (a) (OCR)
The proportion of people who are left-handed is 20%
For a group of 10 students chosen at random, use the binomial distribution to find the probability that
(i)   no student is left handed

[2 marks]

(ii)   exactly 4 students are left handed

[3 marks]

Question 9
Additional Mathematics Examination Question from June 2013, Q6 (OCR)
Amanda throws 3 fair dice.
What is the probability that,
(i)   exactly 2 sixes are thrown

[3 marks]

(ii)   at least 1 six is thrown ?

[3 marks]

Question 10

Additional Mathematics Examination Question from June 2017, Q8 (OCR)

Four ordinary six-sided dice are rolled.

Find the probability that at least 2 sixes are obtained.

[6 marks]