

7.1 Investigating Binomial Probability

Binomial means “two state”: A real life “two state” situation is the flipping of a coin.

Suppose a biased coin is weighted with a probability of 0.36 of landing tails. It is flipped 8 times. What is the probability that it lands tails exactly twice ?

To answer this question we begin by wanting to know how many different ways 2 tails and 6 heads can occur.

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Method 1 : List them and observe the list has 28 entries
 Takes time and is tricky to get correct.

Method 2 : Write out Pascal's Triangle
 Observe that Row 8, Column 2 is 28
 Less time but a tedious job for just one number

1								
1	1							
1	2	1						
1	3	3	1					
1	4	6	4	1				
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	
1	8	28	56	70	56	28	8	1

Method 3 : Use a calculator
 ${}^8C_2 = 28$

Having got the 28...

Each of the 28 items in the list will be a multiplication of $0.36 \times 0.36 \times 0.64 \times 0.64 \times 0.64 \times 0.64 \times 0.64 \times 0.64$ (with those same numbers occurring in different orders)
 Thus the quick calculation is;

$$\begin{aligned}
 & 28 \times 0.36^2 \times 0.64^6 \\
 & = 28 \times 0.1296 \times 0.0687 \\
 & = 0.249 \text{ (Give answers to 3 decimal places)}
 \end{aligned}$$

So, when this biased coin is flipped eight times, there is a 25% probability it will land tails exactly twice.

7.2 Exercise

Marks Available : 60

Question 1

Here is Pascal's Triangle, left justified.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
```

- (i) Circle the entry in Row 7, Column 5
Notice this is in the 8th row and the 6th column !

[1 mark]

- (ii) Use your calculator to determine 7C_5
This should be your part (i) answer

[1 mark]

Question 2

Here is Pascal's Triangle, left justified.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
```

- (i) Circle the entry in Row 4, Column 0

[1 mark]

- (ii) Circle the entry in Row 5, Column 5

[1 mark]

- (iii) Circle the entry in Row 9, Column 3

[1 mark]

- (iv) Find a solution pair (n, r) to

$${}^nC_r = 70$$

[1 mark]

Question 3

Use your calculator to determine ${}^{13}C_5$

[1 mark]

Question 4

A coin is flipped 6 times.

- (i) Describe which calculator buttons you would press in order to determine the number of ways exactly 2 tails could be obtained ?

[1 mark]

- (ii) Write out enough of Pascal's Triangle so that you can then draw a circle around the number in it, corresponding to your part (i) answer

[2 marks]

Question 5

A coin is flipped 20 times.

In how many ways can exactly 13 tails be obtained ?

HINT : Use a calculator.

[2 marks]

Question 6

A coin is flipped 40 times.

In how many ways can exactly 5 heads be obtained ?

[2 marks]

Question 7

A biased coin is weighted such that it has a probability of 0.45 of landing tails. It is flipped 8 times. What is the probability that it lands tails exactly thrice ? Give your answer to 3 decimal places.

[4 marks]

Question 8

A typist has a probability of 0.99 of typing each letter in a sentence correctly. What is the percentage probability of exactly two mistakes in a sentence containing 180 letters, if mistakes are made at random ? Give your answer to 3 decimal places.

[4 marks]

Question 9

In a box of Smarties™ there are eight different colours which normally occur in equal proportions. Sebastian is given 24 Smarties™, and blue ones are his favourite. Assume these come from a very large box.

(i) How many blue Smarties™ would he expect to get (on average) ?

[1 mark]

(ii) What is the probability that he gets this number ?

[4 marks]

(iii) What is the probability that he gets fewer than expected ?

[4 marks]

(iv) What is the probability that he gets more than expected ?

[1 mark]

(v) Explain why the assumption was made that the 24 Smarties™ given to Sebastian came from a very large box ?

[2 marks]

Question 10

(i) Determine 6C_2

[1 mark]

(ii) Determine 6C_4

[1 mark]

(iii) Explain with the help of Pascal's Triangle why ${}^6C_2 = {}^6C_4$

[2 marks]

(iv) Explain, in your own words, why ${}^nC_0 = {}^nC_n$

[2 marks]

(v) Explain, in your own words, why ${}^nC_2 = {}^nC_{n-2}$

[2 marks]

(vi) What formula can be written down for nC_m that generalises the observations made above ?

[2 marks]

The next question is about obtaining the probability distribution curve for a simple coin flipping situation.

Question 11

A biased coin is weighted such that it has a probability of 0.4 of landing tails. It is flipped 6 times.

- (i) Show that the probability of exactly 4 tails being obtained is 0.138. That is, about 14% probable.

[2 marks]

- (ii) Work out the probability of exactly 0, 1, 2, 3, 5 and 6 tails being obtained. Present your solutions in the table below.

N° of tails	0	1	2	3	4	5	6
Probability					14%		

[8 marks]

(iii) Present your table of results as an accurate bar chart.

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk