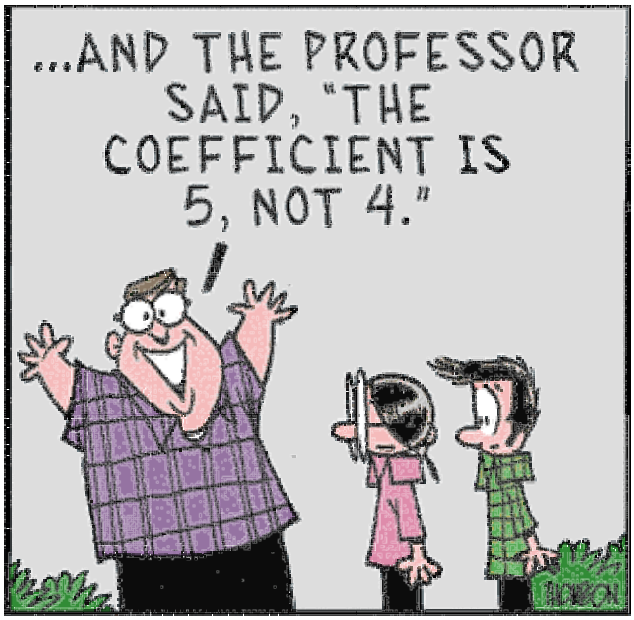
Lesson 9

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

9.1 Coefficient Conundrums



When using the binomial theorem the expansion of the brackets typically leads to a series in ascending powers of x. The number in front of any given power of x is termed the coefficient of that power of x.
For example, consider the expansion;

$$(1 - x)^5 = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

- The coefficient of x^4 is 5
- The coefficient of x^3 is -10

Exam questions sometimes include a puzzle to do with the coefficients.

9.2 Example

When $(a + 3x)^3$ is expanded the coefficient of x is the same as that for x^3
What are the two possible values of the constant a ?

Teaching Video: http://www.NumberWonder.co.uk/v9062/9.mp4

[5 marks]

9.3 Exercise

Marks Available : 40

Question 1

When $(a + 5x)^4$ is expanded the coefficient of x is the same as that for x^3
What are the two possible values of the constant a ?

[5 marks]

Question 2

When $(2 + ax)^3$ is expanded the coefficient of x is the same as that for x^3
What are the two possible values of the constant a ?

[5 marks]

Question 3

A-Level Examination Question from May 2007, Paper C2, Q3 (Edexcel)

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^6$ where k is a non-zero constant.

[5 marks]

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k

[2 marks]

(c) the coefficient of x^3

[1 mark]

Question 4

A-Level Examination Question from June 2009, Paper C2, Q2 (Edexcel)

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of $(2 + kx)^7$ where k is a constant

Give each term in its simplest form

[5 marks]

Given that the coefficient of x^2 is 6 times the coefficient of x

(b) find the value of k

[2 marks]

Question 5

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of $(5 + px)^{30}$ where p is a non-zero constant
There is no need to simplify the terms.

[2 marks]

(b) Given that in this expression the coefficient of x^2 is 29 times the coefficient of x find the value of p

[4 marks]

Question 6

AS Examination Question from May 2018, Q11 (Edexcel)

(a) Find the first 3 terms in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form

[5 marks]

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a

[2 marks]

(c) find the value of b

[2 marks]