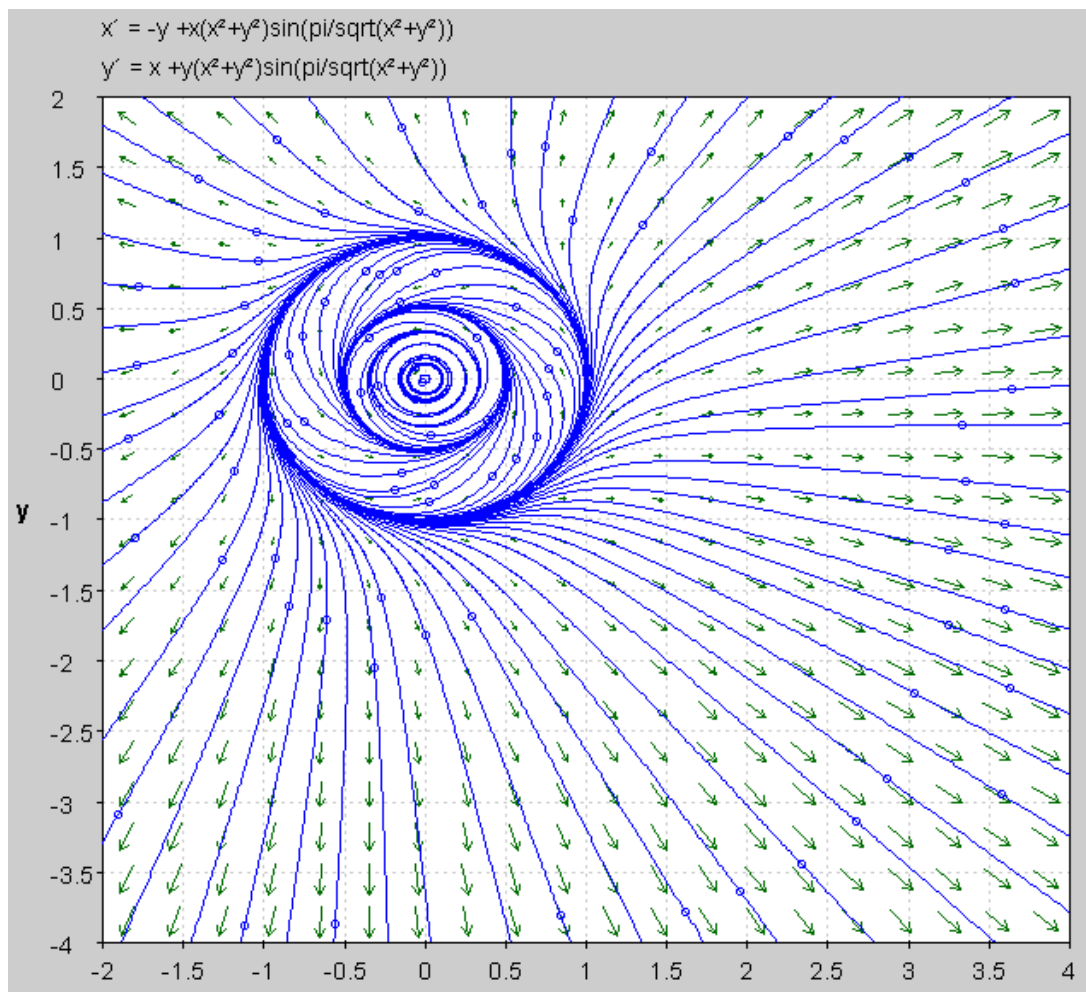


A-Level Pure Mathematics
Year 2

DIFFERENTIAL EQUATIONS I



Differential Equations I

Lesson 1

A-Level Pure Mathematics : Year 2

Differential Equations I

1.1 Type One

Three types of differential equation are to be considered starting in this lesson with the most straight forward.

A Type One differential equation is of the form

$$\frac{dy}{dx} = f(x)$$

These have been encountered before, in the Year 1 pure mathematics course.

Example

A-Level Examination Question from January 2013, Paper C1, Q8 (Edexcel)

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \quad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x , giving each term in its simplest form.

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9066\(1\).mp4](http://www.NumberWonder.co.uk/Video/v9066(1).mp4)

[6 marks]

The Year 2 version of such Type One questions will draw on the fact that it is now known how to integrate many more functions, using many more techniques.

1.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Solve the differential equation,

$$\frac{dy}{dx} = \cos x \quad \text{given that } y = 0.5 \quad \text{when } x = \frac{\pi}{2}$$

[4 marks]

Question 2

Solve the differential equation

$$\frac{dy}{dx} = \sec^2 x \quad \text{given that } y = 1 \quad \text{when } x = \frac{3\pi}{4}$$

[5 marks]

Question 3

- (i) Find the displacement, s cm, from O of a particle at time t s, if its velocity, v cm.s⁻¹, is given by the differential equation

$$v = \frac{ds}{dt} = t^2 - t + 4$$

and the displacement is 100 cm at time 6 s.

[3 marks]

- (ii) What will be the particle's displacement when $t = 3$ s ?

[2 marks]

Question 4

Solve the differential equation,

$$\frac{dy}{dx} = x^2 (x^3 + 5)^4 \text{ given that } y = 209 \text{ when } x = 0$$

Hint : Chain rule backwards.

[5 marks]

Question 5

- (i) Find the general solution of the differential equation

$$5x \frac{dy}{dx} - 1 = 0$$

[4 marks]

- (ii) Given that $y = 0$ when $x = 3$, find the particular solution.
Give your answer in an elegant a form as possible.

[3 marks]

Question 6

- (i) Find the general solution of the differential equation

$$\sec x \frac{dy}{dx} - x = 0$$

Hint : Integration by parts.

[4 marks]

- (ii) Given that $y = \pi$ when $x = \frac{\pi}{2}$, find the particular solution.

[3 marks]

Question 7

Solve the differential equation,

$$\frac{dy}{dx} = \frac{1}{9 + x^2} \quad \text{given that } y = \frac{\pi}{9} \text{ when } x = \sqrt{3}$$

Hint : Let $x = 3 \tan u$

[8 marks]

Question 8

- (i) Find the displacement s m of a particle t s after leaving O , where

$$t \frac{ds}{dt} = t^2 + 4$$

[5 marks]

- (ii) Given that $s = 4 \ln 2$ when $t = 2$, and $s = a + b \ln 2$ when $t = 4$, find a and b .

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk