2.1 Type Two

A Type One differential equation is of the form

$$\frac{dy}{dx} = f(x)$$

A Type Two differential equation is of the form

$$\frac{dy}{dx} = f(y)$$

The solution technique to apply to Type Two differential equation problems involves inverting both sides and then integrating with respect to y
Often, partial fractions arise and the tricky rearranging of a formula.

Example

Solve the following differential equation,

$$\frac{dy}{dx} = \frac{1}{y^4}$$

given that $y = 3$ when $x = 49$
Present your solution in the form $y = f(x)$
Teaching video : http://www.NumberWonder.co.uk/Video/v9066(2).mp4

[6 marks]

2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Solve the following differential equation,

$$\frac{dy}{dx} = \frac{1}{y}$$

given that $y = 3$ when $x = 4$

Present your solution in the form $y = f(x)$

[5 marks]

Question 2

Find the general solution to the differential equation,

$$\frac{dy}{dx} = y$$

Present your solution as elegantly as possible and in the form $y = f(x)$

[5 marks]

Question 3

Find the general solution to the differential equation,

$$\frac{dy}{dx} = y + 2$$

Present your solution as elegantly as possible and in the form $y = f(x)$

[5 marks]

Question 4

Solve the differential equation

$$\frac{dy}{dx} = cos^2 y \qquad \text{given that } y = \frac{\pi}{4} \text{ when } x = 7$$

Present your solution in the form $y = f(x)$

[7 marks]

Question 5

Solve the following differential equation,

$$\frac{dy}{dx} = \frac{1}{2y - 8}$$

given that $y = 5$ when $x = 4$

(i) Present your solution in the form $x = f(y)$

[4 marks]

(ii) By completing the square on your part (i) answer, present your solution in the form $y = f(x)$

[3 marks]

Question 6

A-Level Examination Question from June 2018, Paper C34, Q13 (Edexcel)

(a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.

[2 marks]

The mass, x grams, of a substance at time t seconds after a chemical reaction is modelled by the differential equation,

$$\frac{dx}{dt} = k(4-x)(2-x), \qquad t \geqslant 0,\ 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0,\ x = 0$

(b) solve the differential equation and show that the solution can be written as,

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

[7 marks]

Given that $k = 0.1$

(c) find the value of t when $x = 1$

Giving your answer, in seconds, to 3 significant figures.

[2 marks]