3.1 Type Three

A Type Three differential equation is of the form

$$\frac{dy}{dx} = f(x)\, g(y)$$

The solution technique is called *separating the variables*.

By this is meant rearranging the given equation into the form

$$\frac{1}{g(y)} \frac{dy}{dx} = f(x)$$

and then integrating both sides with respect to x.

Example

Solve the following differential equation,

$$\frac{dy}{dx} = \frac{y^2}{x}$$

given that $y = 4$ when $x = 1$

Present your solution as elegantly as possible and in the form $y = f(x)$

[6 marks]

3.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Solve the following differential equation,

$$\frac{dy}{dx} = 6x\,y^2$$

given that $y = 0.2$ when $x = 1$

Present your solution as elegantly as possible and in the form $y = f(x)$

[6 marks]

Question 2

Solve the following differential equation,

$$\cos^2 x \frac{dy}{dx} - \frac{1}{y^2} = 0$$

given that $y = 2$ when $x = \frac{\pi}{4}$

Present your solution as elegantly as possible and in the form $y = f(x)$

[6 marks]

Question 3

Consider the differential equation;

$$\frac{dy}{dx} = \frac{3x^2}{y} \qquad \text{where } y = 3 \text{ when } x = 2$$

By separating the variables, show that this has solution

$$y^2 = Ax^3 + B$$

where A and B are integers that you should determine.

[4 marks]

Question 4

Consider the differential equation;

$$\frac{dy}{dx} = e^{y+x} \qquad \text{where } y = 0 \text{ when } x = 0$$

By separating the variables, show that this has solution

$$\frac{1}{e^y} + e^x = k$$

where k is an integer that you should determine.

[4 marks]

Question 5

The differential equation

The differential equation $\frac{dv}{dt} = 10 - 0.2v$ models the motion of a body falling vertically subject to air resistance, where v is the downward vertical speed in m/s and the time, t, is in seconds.

(i) Does $\frac{dy}{dx}$ represent displacement, velocity or acceleration ?

[1 mark]

(ii) The body is dropped from rest. What is the initial acceleration ?

[1 mark]

(iii) Find the terminal velocity which occurs when $\frac{dv}{dt} = 0$

[1 mark]

(iv) Show that the differential equation can be written as, $\frac{1}{50 - v}\frac{dv}{dt} = 0.2$

[2 marks]

(v) Remembering that the body was dropped from rest, show that the differential equation can be solved to give that, $v = 50\left(1 - e^{-0.2t}\right)$

[5 marks]

Question 6

(i) Find values of A and B for which, $\dfrac{1}{v(1+v)} = \dfrac{A}{v} + \dfrac{B}{1+v}$

where $v > 0$

[2 marks]

(ii) Show that the differential equation $\dfrac{dv}{dt} = -\left(v + v^2\right)$ where $v = 1$ when $t = 0$ has solution, $\dfrac{2v}{1+v} = e^{-t}$ for $v > 0$

[6 marks]

(iii) Show how the part (ii) answer can be rearranged to give

$$v = \frac{1}{2\,e^t - 1}$$

[2 marks]