Lesson 2

A-Level Applied Mathematics
Mechanics : Moments : Year 2

2.1 Equilibrium

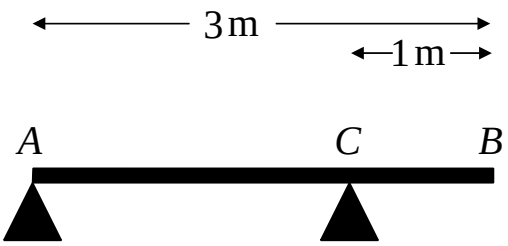
When a rigid body is in equilibrium, the resultant force in any direction is zero and the resultant moment about any point is zero.
In practice, these two general facts are typically is applied as the more specific;

◇ The sum of the forces 'up' must balance the sum of the forces 'down'

◇ The sum of the moments anticlockwise about any pivot equal those clockwise

2.2 Example

An M1 Examination Question from June 2005, Q6



A uniform beam *AB* has mass 12 kg and length 3 m.
The beam rests in equilibrium in a horizontal position, resting on two smooth supports.
One support is at end *A*, the other at a point *C* on the beam, where *BC* = 1 m.
The beam is modelled as a uniform rod.

(a) Find the reaction on the beam at *C*
Teaching video : http://www.NumberWonder.co.uk/Video/v9069(2a).mp4

[3 marks]

A woman of mass 48 kg stands on the beam at the point *D*.
The beam remains in equilibrium.
The reactions on the beam at *A* and *C* are now equal.

(b) Find the distance *AD*
Teaching Video : http://www.NumberWonder.co.uk/Video/v9069(2b).mp4

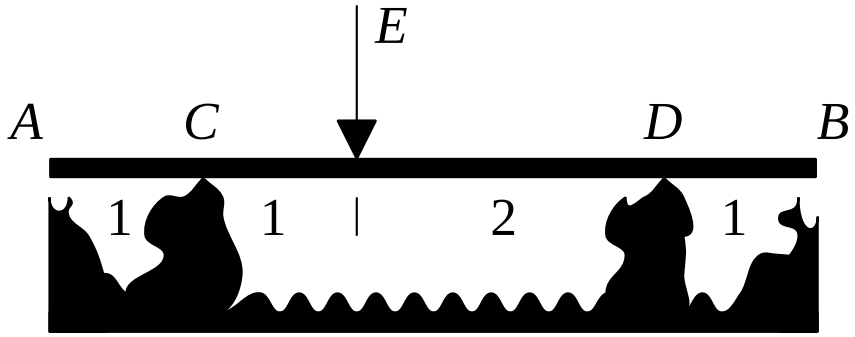
[7 marks]

2.3 Exercise

Question 1

A uniform plank of wood *AB* of length 5 m and mass 30 kg is used to form a bridge across a stream in a forest. The plank of wood rests on two rocks, *C* and *D*, which are each 1 m in from the ends of the plank, as shown.

A lumberjack of mass 75 kg stands on the bridge at *E* where *AE* = 2 m.

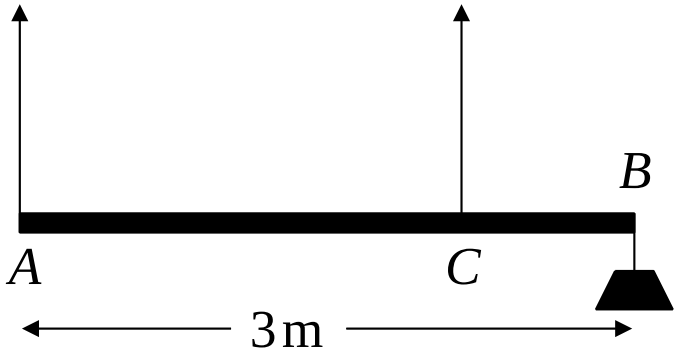


(a) By taking moments about *C*, determine the magnitude of the reaction force at *D*.

(b) By equating the total force upwards with the total force downward determine the reaction force at *C*.

Question 2

M1 Exam question, 12th January 2005, Q2



A plank *AB* has mass 40 kg and length 3 m. A load of mass 20 kg is attached to the plank at *B*. The loaded plank is held in equilibrium, with *AB* horizontal, by two vertical ropes attached at *A* and *C*, as shown. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at *C* is three times the tension in the rope at *A*, calculate

(a) the tension in the rope at *C*

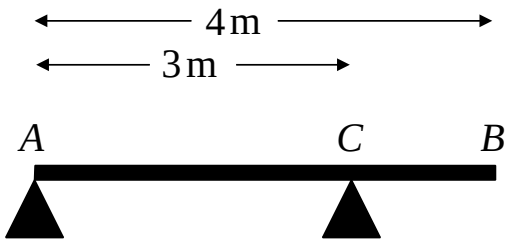
[2 marks]

(b) the distance *CB*

[5 marks]

Question 3

M1 Examination Question, 19th May 2003, Q1



A uniform plank *AB* has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots, one at the end *A*, the other at the point *C* of the plank where *AC* = 3 m.

A man of mass 80 kg stands on the plank which remains in equilibrium.

The magnitudes of the reactions at the two pivots are each equal to *R* newtons.

By modelling the plank as a rod and the man as a particle, find

(a) the value of *R*

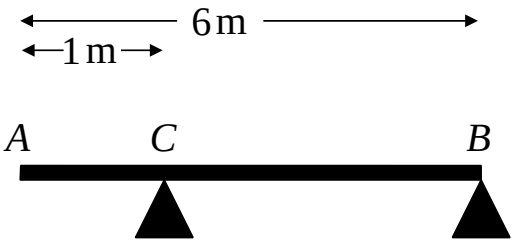
[2 marks]

(b) the distance of the man from *A*

[4 marks]

Question 4

M1 Examination Question, 19th January 2011, Q3



A uniform beam *AB* has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at *C*, where *AC* = 1 m, and the other is at end *B*. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at *B* and at *C*

[5 marks]

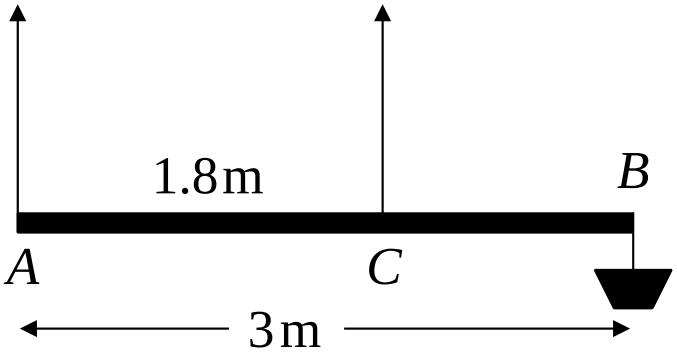
A boy of mass 30 kg stands on the beam at the point *D*.
The beam remains in equilibrium.
The magnitudes of the reactions on the beam at *B* and at *C* are now equal.
The boy is modelled as a particle.

(b) Find the distance *AD*

[5 marks]

Question 5

M1 Exam question, 15th January 2010, Q4



A pole *AB* has length 3 m and weight *W* newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points *A* and *C* where *AC* = 1.8 m, as shown. A load of weight 20 N is attached to the rod at *B*. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at *C* is

$$\left(\frac{5}{6} W + \frac{100}{3} \right) N$$

[4 marks]

(b) Find, in terms of W, the tension in the rope attached to the pole at A.

[3 marks]

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of W.

[3 marks]