Lesson 4

A-Level Applied Mathematics
Mechanics : Moments : Year 2

4.1 The Nonuniform Rod

The assumption that a rod is uniform allowed for its weight to be considered as acting half way along its length. Given a sawn scaffolder's plank of wood it is reasonable to model it as a uniform rod. However, when a tree trunk is used as a bridge, for example, one end of the tree trunk will typically have less circumference than the other; it will have a centre of mass that is more towards the end with the greater circumference. In modelling such a tree trunk with a rod, the rod would be described as *nonuniform*. The centre of mass of the rod is still the place where all of the rod's weight can be considered to act, but that centre of mass is now at an unknown distance along the rod.

4.2 Example

A nonuniform log AB of mass 15 kg and length 8 metres is used as a bridge across a river. The log is supported at P and Q, where AP is 2 metres and QB is 1.5 metres. The reaction at P is twice the reaction at Q.
Find the distance of the centre of mass of the beam from A.
Teaching Video : http://www.NumberWonder.co.uk/Video/v9069(4).mp4

4.3 Exercise

Question 1

A nonuniform bar *AB* of length 5 m has end supports at *A* and *B*
When a man of mass 65 kg stands at the centre of the bar the reactions at supports *A* and *B* are 525 N and 700 N respectively.

(i) Draw a diagram marking on the key forces and lengths.

(ii) By considering all vertical forces, determine the mass of the bar

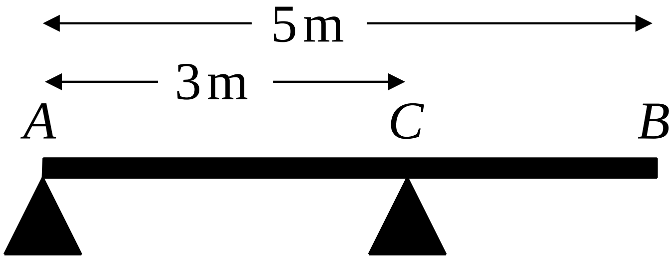
(iii) By taking moments about *A*, find the distance of the centre of mass of the bar from *A*

Question 2

Molly and Mac are sitting on a nonuniform log *AB* of mass 260 kg and length 4 m.
The plank is pivoted at *P*, the midpoint of *AB.*
The centre of mass of *AB* is at *C*, where *AC* is 1.95 m.
Molly has mass 24 kg and sits at *A*.
Mac has a mass of 38 kg.
Where must Mac sit for the log to be in horizontal equilibrium ?

Question 3

A-Level Examination Question from October 2018, IAL, M1, Q2



A nonuniform plank *AB* has weight 60 N and length 5 m. The plank rests horizontally in equilibrium on two smooth supports at *A* and *C*, where *AC* = 3 m, as shown.
A parcel of weight 12 N is placed on the plank at *B* and the plank remains horizontal and in equilibrium. The magnitude of the reaction of the support at *A* on the plank is half the magnitude of the reaction of the support at *C* on the plank.
By modelling the plank as a nonuniform rod and the parcel as a particle,

(a) find the distance of the centre of mass of the plank from *A*

[6 marks]

(b) State briefly how you have used the modelling assumption

(i) that the parcel is a particle

[1 mark]

(ii) that the plank is a rod

[1 mark]

Question 4

A-Level Question from January 2014, IAL, M1, Q3

A beam AB has length 15 m and mass 25 kg. The beam is smoothly supported at the point P, where $AP = 8$ m. A man of mass 100 kg stands on the beam at a distance of 2 m from A and another man stands on the beam at a distance of 1 m from B.

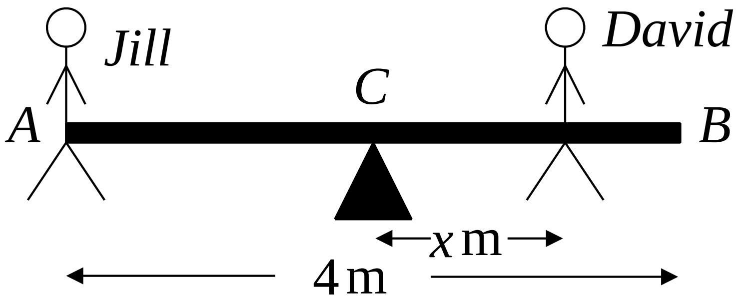
The beam is modelled as a nonuniform rod and the men are modelled as particles.

The beam is in equilibrium in a horizontal position with the reaction on the beam at P having magnitude 2009 N. Find the distance of the centre of mass of the beam from A.

[5 marks]

Question 5

A-Level Question from January 2006, M1, Q3



A seesaw in a playground consists of a beam *AB* of length 4 m which is supported by a smooth pivot at its centre *C*. Jill has a mass 25 kg and sits on the end *A*. David has mass 40 kg and sits at a distance x metres from *C*, as shown. The beam is initially modelled as a uniform rod.

Using this model,

(a) find the value of x for which the seesaw can rest in equilibrium in a horizontal position

[3 marks]

(b) State what is implied by the modelling assumption that the beam is uniform.

[1 mark]

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from *C* for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a nonuniform rod of mass 15 kg.

Using this model,

(c) find the distance of the centre of mass of the beam from *C*.

[4 marks]