5.1 Rods and Simultaneous Equations

The more difficult questions on moments typically involve more algebra, the result of being told less information in a question and so having to work with more unknowns. Often a mass is placed at one location on a rod, an observation made, the mass then moved to another location and another observation made.
The resulting two equations in two unknowns may then be solved either by the method of substitution or as simultaneous equations. Often a question will provide guidance on forming the two equations, but not always.

The two key techniques remain those employed previously, namely that when a rigid body is in equilibrium, the resultant force in any direction is zero (usually the vertical direction) and the resultant moment about any point is zero. Picking the best point to be thought of as a pivot is a skill that develops with experience.
So, when not sure how to proceed, start trying to use the standard techniques.
They remain that,

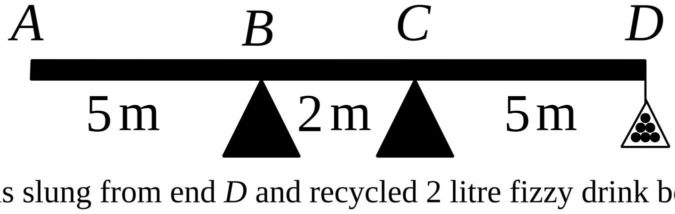
◇ The sum of the forces 'up' must balance the sum of the forces 'down'

◇ The sum of the moments anticlockwise about any pivot equal those clockwise

5.2 How to Weigh a Rod

Surprisingly, the following practical method of determining the mass of a rod without any fancy equipment does not seem to be widely known but provides an excellent introductory example for this lesson.

An aluminium yacht mast *AD* is resting horizontally on two supports *B* and *C* in a boatyard. The mast is 12 metres long. By lifting one end and moving a support, then lifting the other end and moving the other support is is arranged that each support is 5 metres in from an end, with 2 metres between the supports.



A large bag is slung from end *D* and recycled 2 litre fizzy drink bottles are placed one by one into the bag. Once six of these bottles, each full of water, have been added the mast is on the verge of tilting about *C*, as shown.
The large bag is now moved to the other end of the mast. When slung from *A* it takes 18 bottles to cause the mast to be on the verge of tilting about *B*.

(i) How is it immediately known that the mast is nonuniform ?

(ii) What is the mass of the mast ?

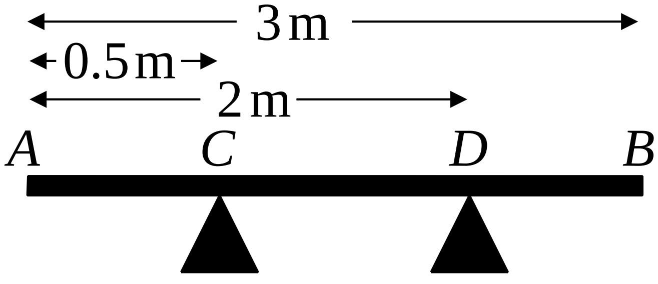
(iii) How far from end *D* is the centre of mass of the mast ?

Over the page, have a go at answering this question before watching the video.

Teaching Video : http://www.NumberWonder.co.uk/Video/v9069(5).mp4

5.3 Exercise

Question 1



A uniform rod *AB* of length 3 m and weight 120 N rests in equilibrium in a horizontal position, smoothly supported at points *C* and *D* where *AC* = 0.5 m and *AD* = 2 m. When a particle of weight *W* newtons is attached at a point *E*, where *AE* = *x* m metres, the rod remains in equilibrium with the reaction at *C* now twice the reaction at *D*.

(i) Draw a diagram marking on all significant forces and lengths.

[2 marks]

(ii) By taking moments about *A*, show that $3R_D = Wx + 180$

[2 marks]

(iii) By balancing forces vertically, show that $3R_D = 120 + W$

[2 marks]

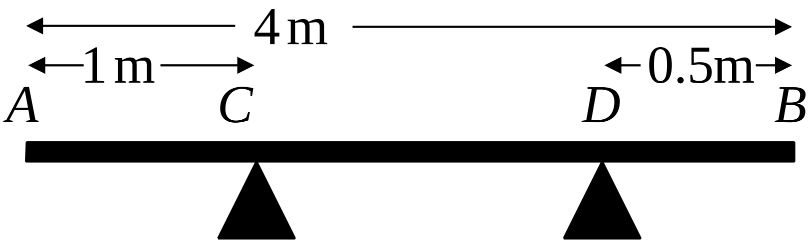
(iv) Hence show that $W = \dfrac{60}{1 - x}$

[2 marks]

(v) Hence deduce the range of possible values of *x*

[2 marks]

Question 2



A nonuniform rod AB has length 4 m and weight 150 N. The rod rests horizontally in equilibrium on two smooth supports C and D, where AC = 1 m and DB = 0.5 m. The centre of mass of AB is x metres from A. R_C and R_D are the magnitudes of the reactions at C and D respectively. A particle of weight W N is placed on the rod at A. The rod remains in equilibrium with R_C then 100 N.

(i) Draw a diagram marking on all significant lengths and forces.

[2 marks]

(ii) By balancing forces vertically show that $R_D = W + 50$

[1 mark]

(iii) By taking moments about A and then using your part (i) answer, show that $300x = 550 + 7W$

[4 marks]

The particle is now removed from *A* and placed on the rod at *B*. The rod remains in equilibrium and the reaction of *C* on the rod, R_C', now has magnitude 52 N.

Note that the reaction at *D*, R_D' will also now have changed.

(iv) Repeat steps (i), (ii) and (iii) to show that $300x = 790 + 7W$

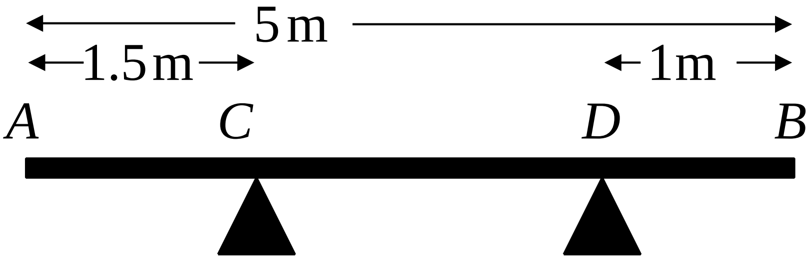
[4 marks]

(v) Combine the equations obtained previously to calculate the value of *x* and the value of *W*

[3 marks]

Question 3

M1 Examination Question from November 2003, Q6



A nonuniform rod *AB* has length 5 m and weight 200 N. The rod rests horizontally in equilibrium on two smooth supports *C* and *D*, where *AC* = 1.5 m and *DB* = 1 m. The centre of mass of *AB* is *x* metres from *A*.

A particle of weight *W* N is placed on the rod at *A*. The rod remains in equilibrium and the magnitude of the reaction of *C* on the rod is 160 N.

(a) Show that $50x - W = 100$

[5 marks]

The particle is now removed from *A* and placed on the rod at *B*. The rod remains in equilibrium and the reaction of *C* on the rod now has magnitude 50 N

(b) Obtain another equation connecting W and x

[3 marks]

(c) Calculate the value of x and the value of W

[4 marks]

Question 4

M1 Examination Question from June 2016, Q6

A nonuniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where AS = 0.5 m and TB = 2 m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tipping about S.

When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tipping about T.

The distance of the centre of mass of the plank from A is d metres.
The block is modelled as a particle and the plank is modelled as a nonuniform rod.

Find **(i)** the value of d

(ii) the value of M

[7 marks]