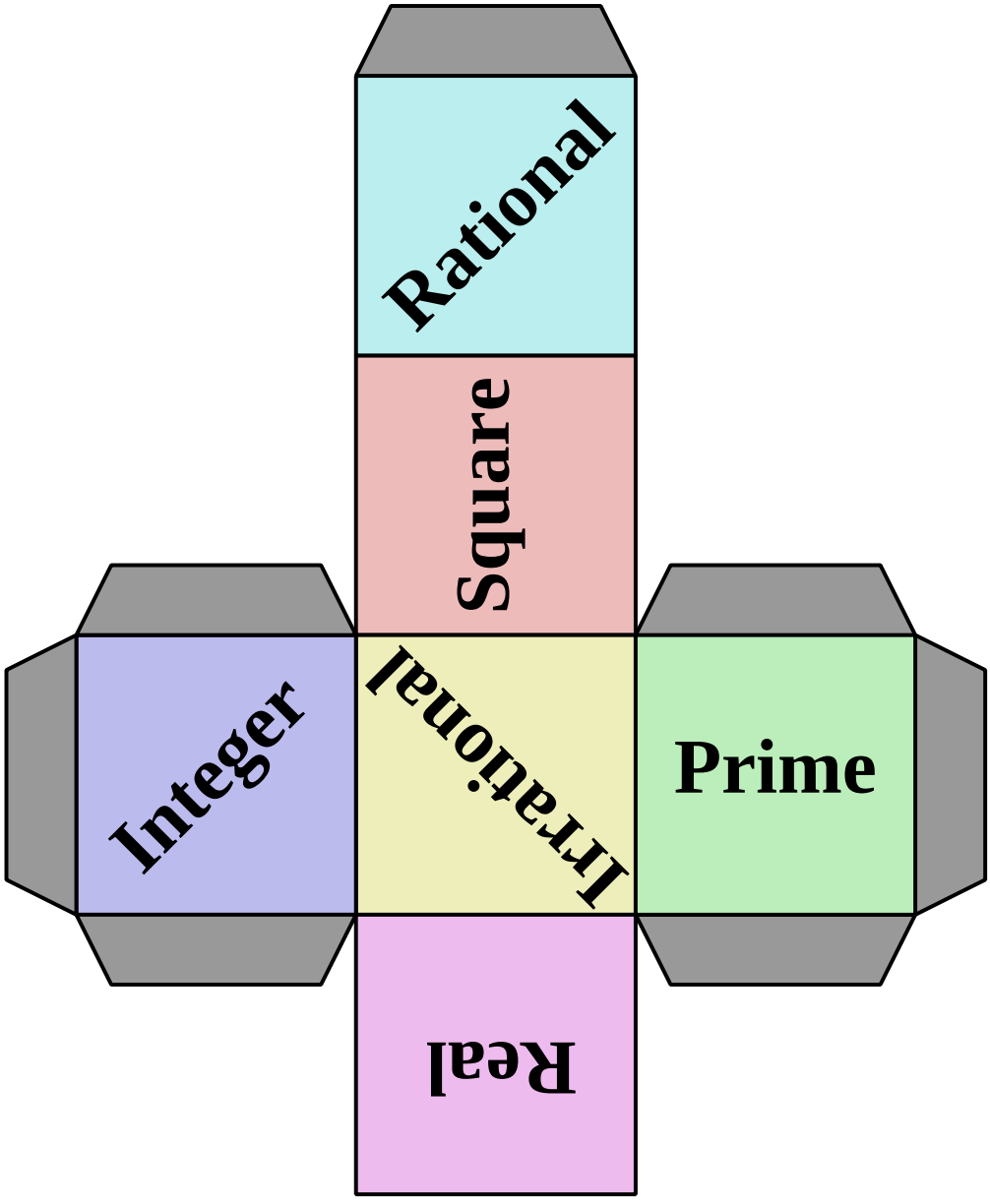
The Classification of Numbers



The Classification of Numbers

Chapter 1

GCSE Mathematics
The Classification of Numbers

1.1 The Integers

The integers are the numbers in the set,

$$\mathbb{Z} = [\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

A mathematician who's is primarily interested in integers is called a Number Theorist. This lesson is a reminder of some of the many pieces of GCSE mathematics that use only the integers. The statement, $n \in \mathbb{Z}$, states that "n is an integer".

1.2 Exercise

Do ***NOT*** use a calculator
Instead, use your mind !
Marks Available : 70

Question 1

List the integers which satisfy the following "sandwich" inequalities:

(i) $21 \leqslant n < 23$

[1 mark]

(ii) $3.2 < n \leqslant 6.8$

[1 mark]

(iii) $-2.13 < n < 0.75$

[1 mark]

(iv) $\sqrt{16} \leqslant n < \sqrt{81}$

[1 mark]

(v) $2\pi < n < 3\pi$

[1 mark]

(vi) $n^2 \leqslant 9$ (Hint: There are 7 integers that make this true)

[2 marks]

Question 2

The positive integers, with the exception of the number 1, are one of either "prime" or "composite".

For each of the following numbers state if the number is prime or composite;

(i)	7	**(ii)**	27	**(iii)**	1082
(iv)	43	**(v)**	91	**(vi)**	101
(vii)	234785	**(viii)**	23	**(ix)**	2

[9 marks]

Question 3

List the integers which satisfy the following "sandwich" inequalities:

(i) $32 < 3n + 8 \leqslant 41$

[2 marks]

(ii) $13 \leqslant 4n + 5 \leqslant 21$

[2 marks]

(iii) $4 < 2n - 1 \leqslant 10$

[2 marks]

(iv) $0.2 \leqslant 3n + 0.5 \leqslant 3.8$

[2 marks]

(v) $-3 < 7n - 3 < 25$

[2 marks]

Question 4

Demonstrate how to find the six digit value of $2^4 \times 3^3 \times 5^4$ without a calculator.

[3 marks]

Question 5
Prof Iama Duffer has a theory:

- Take any even number.
- Square it.
- Add one.

- The result is always a prime number.

Prof Iama Duffer is wrong.
Give an example that ***proves*** he is wrong.

[2 marks]

Question 6
Explain why the number 187 is NOT a prime number.

[2 marks]

Question 7

The number 60 can be written as a product of prime numbers like this:

$$60 = 2^2 \times 3 \times 5$$

Demonstrate how to write the number 588 as a product of prime numbers without the use of a calculator.

[3 marks]

Question 8
You are told the fact that $9 \times 9 \times 9 \times 9 = 6561$
Explain how to use this fact to find the square root of 6561 without using a calculator.

[2 marks]

Question 9

The Lowest Common Multiple (*lcm*) of 14 and 21 is the first number that is in the 14 times table and the 21 times table.
What is the *lcm* of 14 and 21 ?

[2 marks]

Question 10

Show how to find the *lcm* of 15 and 35 without using a calculator.

[3 marks]

Question 11

Complete the bottom two rows of the following arithmetical triangle.

$$\begin{aligned} 1 &= 2^0 \\ 1 + 1 &= 2^1 \\ 1 + 2 + 1 &= 2^2 \\ 1 + 3 + 3 + 1 &= 2^3 \\ 1 + 4 + 6 + 4 + 1 &= 2^4 \\ \ldots &= \ldots \end{aligned}$$

This triangular array of numbers is world famous.
What is it's name ?

[1 mark]

Question 12

What is the Highest Common Factor (*hcf*) of 40 and 56 ?

[2 marks]

Question 13

Two numbers are ***coprime*** when they have no prime factors in common.
(In other words, when the *hcf* of the two numbers is 1)
For each of the following integer pairs state if they are coprime or not.

(i) 7 and 11 **(ii)** 15 and 21 **(iii)** 49 and 52

(iv) 10 and 11 **(v)** 13 and 39 **(vi)** 63 and 56

[6 marks]

Question 14

(i) Use a calculator with a "FACT" button to write 104 and 1375 as products of primes (The "FACT" is short for factorise).

[2 marks]

(ii) Hence, explain why 104 and 1375 are coprime.

[2 marks]

(iii) Find the *lcm* of 104 and 1375
(Hint: You may have noticed in question 13 that when two numbers are coprime their *lcm* is the product of the two numbers)

[2 marks]

Question 15

Without using a calculator determine the value of;

(i) 16^2 **(ii)** 16^0 **(iii)** 16^1

(iv) $16^{0.5}$ **(v)** 16^{-1} **(vi)** $16^{-0.25}$

[6 marks]

Question 16

(i) Without using a calculator, write 7056 as a product of primes.

[4 marks]

(ii) Use your part **(i)** answer to square root 7056

[2 marks]

* ***Did you know ?***

The number 2 is the ONLY even prime.
Which makes it rather odd. (Ha, ha !)