

Chapter 2

GCSE Mathematics The Classification of Numbers

2.1 The Rationals

A *rational* number is a number that can be written in the form

$$\frac{p}{q}$$

where p and q are integers and $q \neq 0$.

Like the integers, the rationals can be methodically listed,

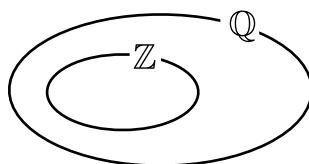
$$\mathbb{Q} = \left\{ \dots, -\frac{2}{3}, -\frac{3}{2}, -\frac{4}{1}, -\frac{3}{1}, -\frac{1}{3}, -\frac{1}{2}, -\frac{2}{1}, -\frac{1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{1}{3}, \frac{3}{1}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \dots \right\}$$

The statement, $n \in \mathbb{Q}$, states that “ n is a rational number”.

In the list of rationals, some entries are highlighted in red.

Those entries were the integer values.

This illustrates the fact that all numbers that are integers, are also rationals.



The GCSE examination often has a question that gives candidates a number written as a decimal fraction and the task is to show that it is a rational number by writing it in the form,

$$\frac{p}{q} \quad p, q \in \mathbb{Z}, q \neq 0$$

2.2 Terminating Decimal Example

Show that $0.125 \in \mathbb{Q}$, giving your answer in as simple a form as possible.

[2 marks]

2.3 Repeating Pattern Decimal Example

Show that $0.63\ 63\ 63\ 63\ 63\ 63\dots$ is rational.

Give your answer in as simple a form as possible.

[3 marks]

2.4 Exercise

Do *NOT* use a calculator

Marks Available : 50

Question 1

Show that 0.225 is rational by writing it in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

Simplify your answer if it is possible to do so.

[2 marks]

Question 2

Consider the number, $0.\dot{3}\ddot{6}$

Show that this number is rational by writing it in the form $\frac{p}{q}$ for integer p and q

and with $q \neq 0$. Simplify your answer if it is possible to do so.

[3 marks]

Question 3

Consider the number, $0.\dot{7}\ddot{5}$

Show this is rational by writing it in the form $\frac{p}{q}$ for $p, q \in \mathbb{Z}$, $q \neq 0$.

Simplify your answer if it is possible to do so.

[3 marks]

Question 4

Show that the following number is rational;

$$0.\dot{3}5\dot{1}$$

Simplify your answer if it is possible to do so.

[3 marks]

Question 5

(i) Write 0.875 in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

[2 marks]

(ii) What one of the following is true ?

- 0.875 is a natural number, \mathbb{N}
- 0.875 is an integer, \mathbb{Z}
- 0.875 is a positive integer, \mathbb{Z}^+
- 0.875 is a rational number, \mathbb{Q}

[1 mark]

Question 6

Write 2.52 in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

Simplify your answer if it is possible to do so.

[2 marks]

Question 7

(i) Write the integer 23 in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

[1 mark]

(ii) Explain why all integers are also rational numbers.

[2 marks]

Question 8

Show that the infinitely repeating decimal $0.\dot{1}463\dot{4}$ is a member of the set of rational numbers, \mathbb{Q} , by writing it in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$.

Give your answer in simplified form.

[5 marks]

Question 9

(i) Show that $0.44444444 \dots \in \mathbb{Q}$

[2 marks]

(ii) Show that any decimal of the form $0.aaaaaaaa\dots$ is rational.

[5 marks]

Question 10

(i) Show that the decimal $0.122222222 \dots$ is rational

[2 marks]

(ii) Show that any decimal of the form $0.1aaaaaaaa\dots$ is rational

[5 marks]

Question 11

Show that any decimal of the form $0.ab\ ab\ ab\ ab\ ab\ ab\ \dots$ is rational

[5 marks]

Question 12

Prove that the decimal;

$0.\dot{0}2197\bar{8}$

is a rational number.

[4 marks]

Question 13

Prove that $0.999999999999... = 1$

[3 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk