4.1 π

This world famous number is defined as being the ratio of ***any*** circle's circumference, *C*, to its diameter, *d*;

$$C : d$$

In other words,

$$\pi = \frac{C}{d}$$

No matter what circle is having its circumference and diameter measured, when the first is divided by the second, the same number, π, is the result.
Like $\sqrt{2}$, π is an irrational number, $\mathbb{P}$, and so it can not be written ***exactly*** as a ratio of integers, or exactly as a decimal.

$$\pi = 3.14\ldots \ (approximately)$$

The value of π is stored to more decimal places on your calculator. A practical use of knowing π, and of knowing that it is the same for all circles, is that a couple of useful formulae now allow us to measure a particular circle's radius, (which is easy from a practical point of view) and calculate its area, *A*, and its circumference, *C*, (which are awkward to do by other means).

$$A = \pi r^2$$

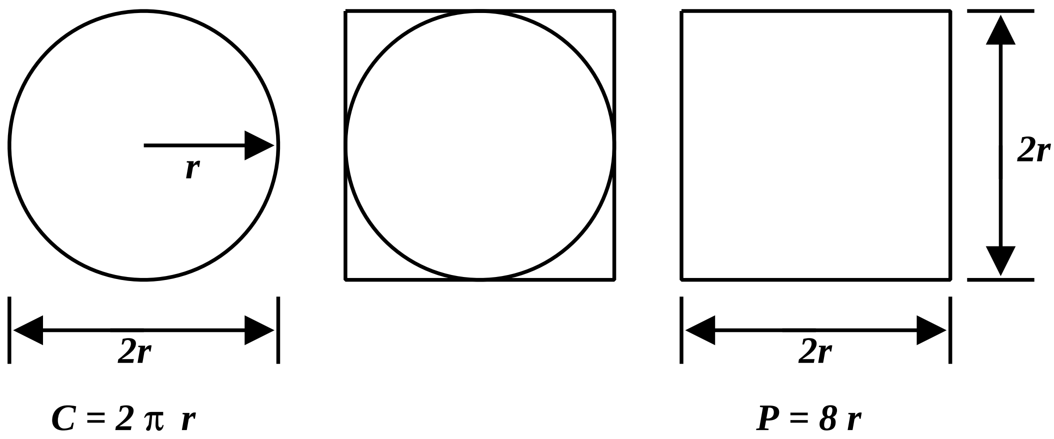
and

$$C = 2\pi r$$

4.2 Proof that π is not an integer

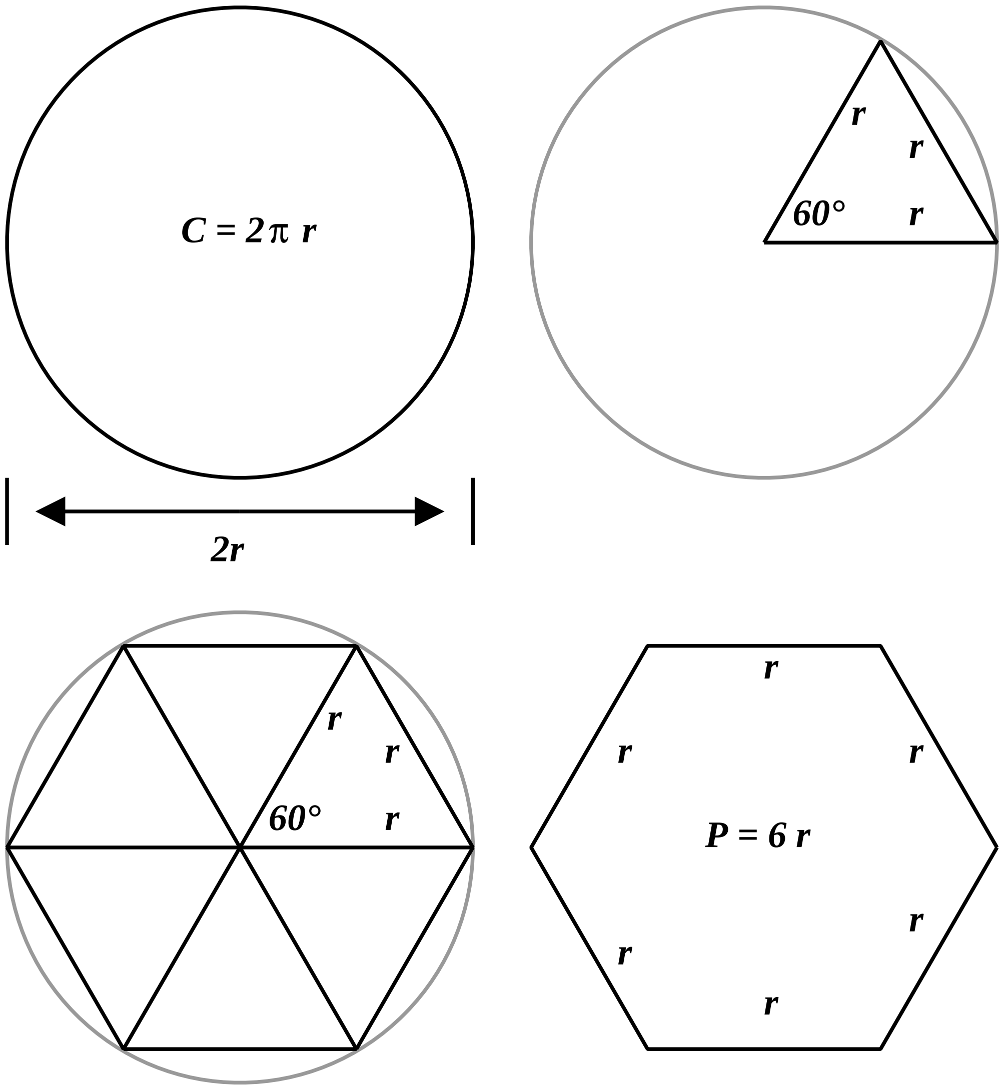
It is difficult to prove that π is irrational, but we can easily prove it's not an integer. The proof is in two parts.

Firstly, consider the following where a circle is placed inside a square,



$$C < 8r$$

Secondly, consider the following where a hexagon is placed inside a circle,



$$C \; > \; 6r$$

So, we have a sandwich inequality,

$$6r \; < \; C \; < \; 8r$$

$$6r \; < \; 2\pi r \; < \; 8r$$

$$6 \; < \; 2\pi \; < \; 8 \qquad \text{(Dividing through by } r\text{)}$$

$$3 \; < \; \pi \; < \; 4 \qquad \text{(Dividing through by 2)}$$

As there are no integers between 3 and 4 and we have a mathematical proof (a watertight argument) that π can not be an integer, □

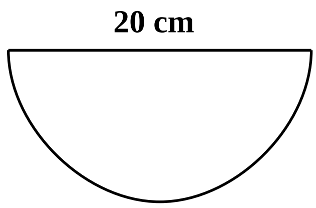
4.3 Exercise

You may use a calculator

Marks Available : 60

Question 1

A bathroom towel holder is made from a length of wire in the shape of a semi-circle of diameter 20 cm.



Calculate the length of the wire used.
Give your answer correct to two decimal places.

[3 marks]

Question 2

Which of the following is the closest approximation to π ?

$\frac{25}{8}$ $\qquad$ $\frac{3}{1}$ $\qquad$ $\sqrt{10}$

[2 marks]

Question 3

Which of the following is the closest approximation to $\sqrt{2}$?

$\frac{3}{2}$ $\qquad$ $\frac{\pi}{2}$ $\qquad$ $\frac{7}{5}$

[2 marks]

Question 4

Under each expression write $\mathbb{Q}$ if it's *rational*, or $\mathbb{P}$ if it's *irrational.*

(i) π^2 **(ii)** $\dfrac{8\pi}{3\pi}$

(iii) $\sqrt{\pi}$ **(iv)** π^0

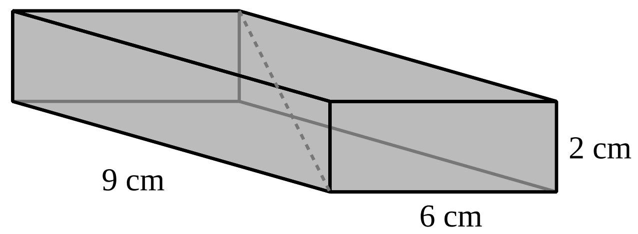
(v) $\dfrac{1}{\pi}$ **(vi)** $(\pi + 5)(\pi - 5) - \pi^2$

[6 marks]

Question 5

In a cuboid with sides of lengths *a*, *b* and *c*, the longest diagonal in the box, *d*, is given by a three dimensional version of Pythagoras' Theorem.

$$d^2 = a^2 + b^2 + c^2$$



(i) Work out the length of the longest diagonal of a cuboid that measures 9 cm by 6 cm by 2 cm

[2 marks]

(ii) Is your part **(i)** answer a rational number or an irrational number ?

[1 mark]

Question 6

Although $\sqrt{12}$ is an irrational number, and $\sqrt{3}$ is also an irrational number, the expression $\dfrac{7\sqrt{12}}{2\sqrt{3}}$ is rational. Explain why this is so.

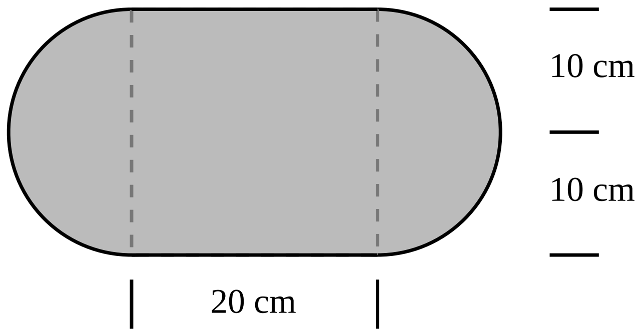
[3 marks]

Question 7

(i) Find the AREA of a circle of radius 10 cm.
Give your answer correct to the nearest integer.

[2 marks]

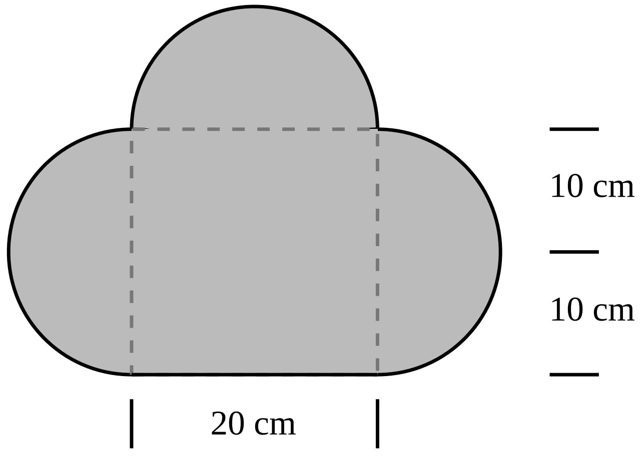
(ii)



Find the AREA of the shape.
Give your answer to the nearest integer.

[2 marks]

(iii)



Find the AREA of the shape.
Give your answer to the nearest integer.

[2 marks]

Question 8

GCSE Examination Question from June 1995, Q11 (Edexcel)

$$x = \sqrt{a^2 + b^2}$$

State whether x is ***rational*** or ***irrational*** in each of the following cases, and show sufficient working to justify each answer.

(a) $a = 5$ and $b = 12$

[2 marks]

(b) $a = 5$ and $b = 6$

[2 marks]

(c) $a = \sqrt{2}$ and $b = \sqrt{7}$

[2 marks]

(d) $a = \dfrac{3}{7}$ and $b = \dfrac{4}{7}$

[2 marks]

Question 9

GCSE Examination Question from June 1996, Q8 (Edexcel)

Here are some ***irrational*** numbers.

$$\sqrt{3} \qquad \sqrt{5} \qquad \sqrt{8} \qquad \pi \qquad \sqrt{12} \qquad \sqrt{50}$$

(a) Use two of these numbers to show that, if two ***irrational*** numbers are multiplied together, the result can be a ***rational*** number.

[2 marks]

(b) Use two of these numbers to show that, if two ***irrational*** numbers are divided, the result can be a ***rational*** number.

[2 marks]

Question 10

GCSE Examination Question from June 1994, Q7 (Edexcel)

Which of the following numbers are ***rational*** and which ***irrational*** ?

$$\sqrt{4\frac{1}{4}} \qquad \sqrt{6\frac{1}{4}} \qquad \frac{1}{3} + \sqrt{3} \qquad \left(\frac{1}{3}\sqrt{3}\right)^2$$

Express each of the ***rational*** numbers in the form $\frac{p}{q}$ where p and q are integers, $q \neq 0$.

[5 marks]

Question 11

Prove that each of the following statements is false by giving a counter example.

(i) The square of ***any number*** is always greater than the original number.

[2 marks]

(ii) For every pair of ***integers***, n and m, if $n > m$ then $n^2 > m^2$

[2 marks]

(iii) The product of two ***irrational*** numbers is always irrational.

[2 marks]

Question 12

Under each expression write $\mathbb{Q}$ if it's *rational*, or $\mathbb{P}$ if it's *irrational.*

(i) $\sqrt{2}$ **(ii)** $\sqrt{2} \times \sqrt{8}$ **(iii)** $8 \times \sqrt{2}$

(iv) $\sqrt{8 + 2}$ **(v)** $\dfrac{\sqrt{8}}{\sqrt{2}}$ **(vi)** $\sqrt{8}$

[6 marks]

Question 13

If $a = 1 + \sqrt{2}$ and $b = 1 - \sqrt{2}$ find the ***exact*** value of the following;
(You will have to leave irrational numbers written as square roots)
In each case state if your answer is ***Rational*** or ***Irrational***.

(i) $a + b$ **(ii)** $a - b$ **(iii)** $2a + b$

(iv) $\sqrt{2}\, a$ **(v)** a^2 **(vi)** ab

[6 marks]