Chapter 5

GCSE Mathematics

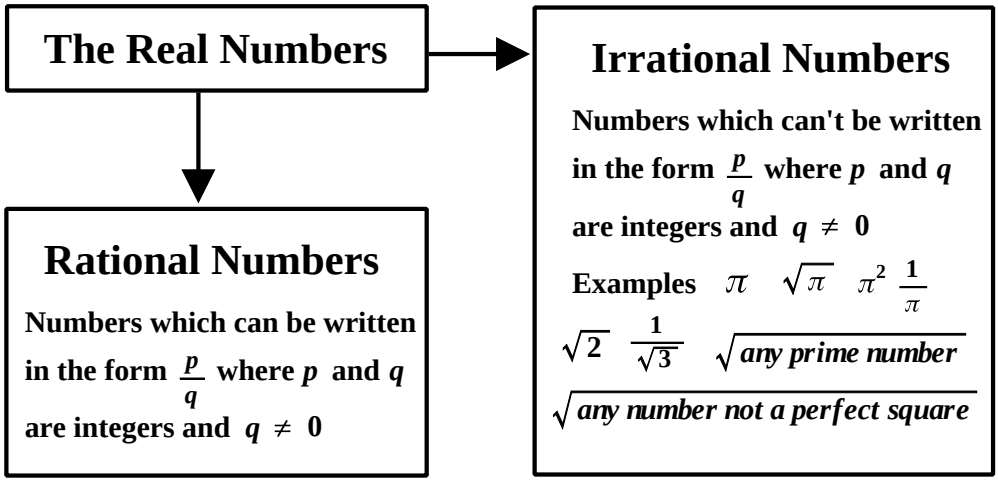
The Classification of Numbers

5.1 Revision

Make sure you know the difference between,

- ***Integers*** $\mathbb{Z}$
- ***Rationals*** $\mathbb{Q}$
- ***Irrationals*** $\mathbb{P}$

If all of the rational numbers, $\mathbb{Q}$, are merged with all of the irrational numbers, $\mathbb{P}$, the result is the real numbers, $\mathbb{R}$, which (for GCSE) are "all the numbers you know about" (At A-Level another type of number is encountered).



Integers: $\mathbb{Z}$ Numbers from the set { ... -3, -2, -1, 0, 1, 2, 3, ... }
If it helps, think of these as the "not fractions"
$\left(\text{But keep in mind that, for example, } 7 = \frac{7}{1}\right)$

Rationals: $\mathbb{Q}$ Numbers that can be written in the form $\frac{p}{q}$ for integer p and q.

Note that $q \neq 0$, as division by zero has no mathematical meaning.
When written as a decimal expansion the digits of the expansion
either * terminate
or * form a repeating pattern without end

Irrationals: $\mathbb{P}$ Numbers like π and $\sqrt{2}$

Such numbers CANNOT be written as $\frac{p}{q}$ for integer p and q.

When written as a decimal expansion the digits of the expansion
* never terminate
and * form no repeating pattern

Exam questions often try to write a rational number in a way that makes it look like an irrational number,

$$(\sqrt{\pi})^0$$

or make an irrational number look like it is rational,

$$\frac{\pi + 1}{\pi}$$

5.2 Exercise

Do NOT use a calculator

Marks Available : 50

Question 1

$\sqrt{7}$ = 2.645751311 ...

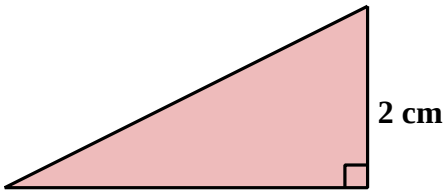
List all of the integers, n, which satisfy

$$-\sqrt{7} < n < \sqrt{7}$$

[2 marks]

Question 2

Calculate the ***exact*** length of this triangle's hypotenuse.



[3 marks]

Question 3

(i) Leaving your answer written in terms of π, calculate the ***exact*** area of a circle which has a radius of 12 cm.

[3 marks]

(ii) Give your part (i) answer as a decimal accurate to one decimal place.

[1 mark]

Question 4

A helpful table of squares:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

By using the helpful table of squares, or otherwise, determine whether each of the following numbers is ***Rational***, $\mathbb{Q}$, ***Irrational***, $\mathbb{P}$.

(i) $\sqrt{256}$

(ii) $\sqrt{291}$

(ii) $\sqrt{10^2 + 5^2 + 10^2}$

[3 marks]

Question 5

Show that each of the following is rational by writing them in the form

$$\frac{p}{q}$$

where p and q are integers, $q \neq 0$.

(i) $\left(\frac{7}{9}\right)^2$

[1 mark]

(ii) $\sqrt{\frac{100}{289}}$

[1 mark]

(iii) $\left(3 + \frac{1}{4}\right)^2$

[2 marks]

Question 6

$\sqrt{5}$ = 2.236067977 ...

$\sqrt{6}$ = 2.449489743 ...

Write down a rational number that lies between $\sqrt{5}$ and $\sqrt{6}$

[2 marks]

Question 7

For each expression state if it is ***Rational***, $\mathbb{Q}$, or ***Irrational***, $\mathbb{P}$.
Show working where necessary.

(i) $(\sqrt{3})^2$ **(ii)** $\frac{\sqrt{20}}{\sqrt{4}}$

(iii) 1^{π} **(iv)** $(\sqrt{7})^0$

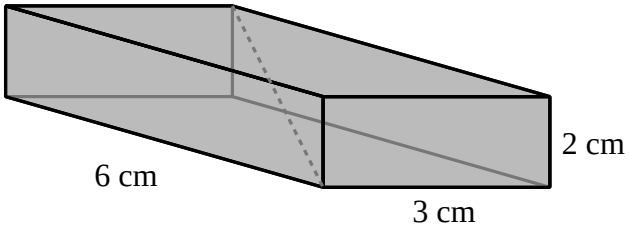
(v) $5^{0.5}$ **(vi)** $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

[6 marks]

Question 8

In a cuboid with sides of lengths *a*, *b* and *c*, the longest diagonal in the box, *d*,
is given by a three dimensional version of Pythagoras' Theorem.

$$d^2 = a^2 + b^2 + c^2$$



(i) Work out the length of the longest diagonal of a cuboid that measures
6 cm by 3 cm by 2 cm

[2 marks]

(ii) Is your part **(i)** answer a rational number, $\mathbb{Q}$, or an irrational number, $\mathbb{P}$?

[1 mark]

Question 9

Show that 0.625 can be written in the form $\frac{p}{q}$ for $p, q \in \mathbb{Z}$ and $q \neq 0$.

Simplify your answer if it is possible to do so.

[3 marks]

Question 10

Victor says that as $\pi = \frac{22}{7}$, π must be a rational number.

Explain his error.

[2 marks]

Question 11

List the integers, if any, which satisfy

$$29 \leqslant 3n + 14 < 38$$

[3 marks]

Question 12

Consider the number $0.\dot{4}\ddot{5}$

That is, 0.454 545 454 545 ...

Show how this number can be rewritten in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

Simplify your answer if it is possible to do so.

[3 marks]

Question 13

Show that each of the following is rational by writing them in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $\left(\frac{1}{2} + \frac{1}{3}\right)^2$ **(ii)** $\sqrt{\frac{3}{5} + \frac{1}{25}}$

[4 marks]

Question 14

Use the theorem of Pythagoras to show that a right angled triangle with shorter sides of lengths $\frac{6}{5}$ cm and $\frac{8}{5}$ cm has a hypotenuse of integer length.

[3 marks]

Question 15

Consider the number

$$0.041\dot{6}$$

Let $x = 0.0416666666666666.....$

(i) Work out the value of $10\,000\,x$

[1 mark]

(ii) Work out the value of $1\,000\,x$

[1 mark]

(iii) By subtracting answer **(b)** from answer **(a)** work out $9\,000\,x$

[1 mark]

(iv) Hence write $0.041\dot{6}$ in the form of a rational number.

That is, in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Simplify your answer if it is possible to do so.

[2 marks]