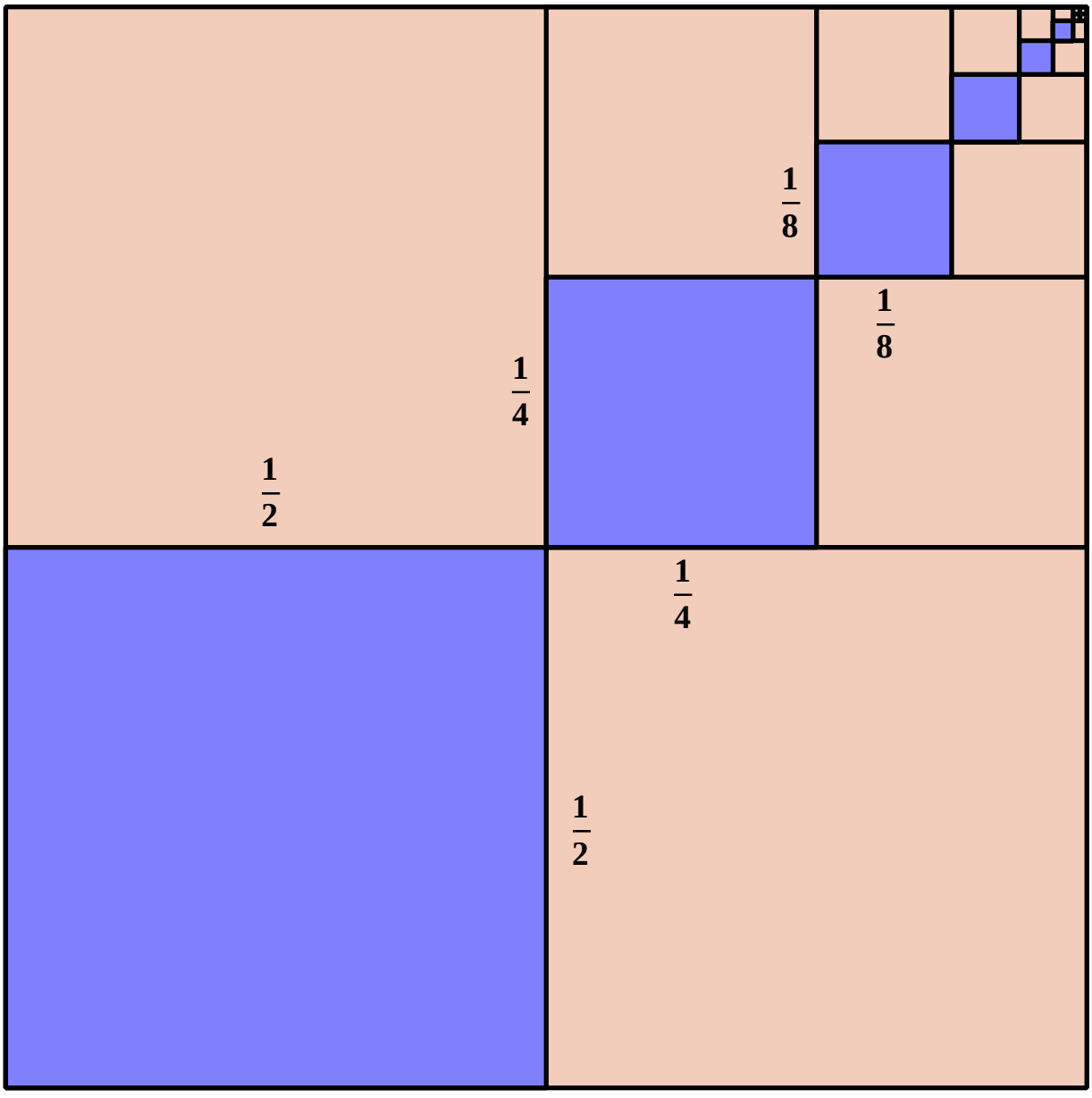
A-Level

~ Year 2 ~

Pure Mathematics

GEOMETRIC PROGRESSIONS



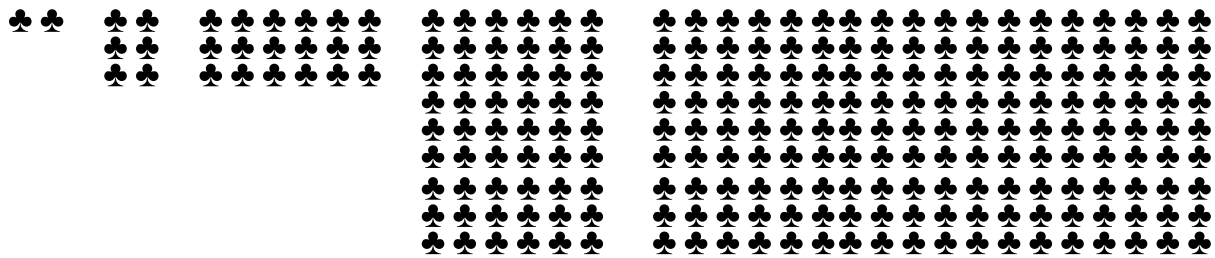
$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \frac{1}{4096} + \ldots = \frac{1}{3}$$

Geometric Progressions

Lesson 1

A-Level Pure Mathematics, Year 2
Geometric Progressions

1.1 How To Spot A Geometric Progression



Consider the sum

$$2 + 6 + 18 + 54 + 162 + \ldots$$

Explain why this series not an Arithmetic Progression

☞

[1 mark]

Teaching Video : http://www.NumberWonder.co.uk/v9077/1.mp4

Observe that the terms are linked; each is three times the previous.
This is the hallmark of a Geometric Progression.
In this case it is said that the common ratio is 3

Expressed algebraically, a Geometric Progression is of the form

$$a, \quad ar, \quad ar^2, \quad ar^3, \quad ar^4, \quad \ldots$$

where a is the first term
and r is the common ratio

Write down a formula for the n^{th} term, G_n of a Geometric Progression

☞

[1 mark]

1.2 Example

The 5^{th} term if a Geometric Progression is 567 and the 2^{nd} term is 21

(i) What is the common ratio ?

[3 marks]

(ii) Write out the first 6 terms of the Geometric Progression.

[2 marks]

(iii) Determine the exact value of the 20^{th} term.

[2 marks]

1.3 Exercise

Marks Available: 60

Question 1

Write out the first five terms of the Geometric Progression with first term 8 and common ratio 1.5

[2 marks]

Question 2

Write out the first five terms of the Geometric Progression with first term 8 and common ratio 0.5

[2 marks]

Question 3

Write out the first five terms of the Geometric Progression with first term 3 and common ratio -2

[2 marks]

Question 4

Consider the following Geometric Progression;

$$0.3, \quad 0.03, \quad 0.003, \quad 0.0003, \quad ...$$

(i) State the value of the first term, a, and the value of the common ratio, r

[2 marks]

The sum of this Geometric Progression has an infinite number of terms

$$0.3 + 0.03 + 0.003 + 0.0003 + ...$$

This infinite sum has a finite answer.

(ii) Give the exact value of this "sum to infinity"

[2 marks]

Question 5

What is the exact value of the 20^{th} term of the following Geometric Progression ?

$$5, \quad 15, \quad 45, \quad 135, \quad ...$$

[3 marks]

Question 6

The 5^{th} term of a Geometric Progression is 3750 and the 2^{nd} term is 30

(i) What is the common ratio ?

[3 marks]

(ii) Write out the first 6 terms of the Geometric Progression.

[2 marks]

(iii) Determine the exact value of the 12^{th} term.

[2 marks]

Question 7

The 6^{th} term of a Geometric Progression is 0.375 and the 3^{rd} term is – 3

(i) What is the common ratio ?

[3 marks]

(ii) Write out the first 6 terms of the Geometric Progression.

[2 marks]

(iii) Determine the exact value of the 20^{th} term.

Write your answer as a $\frac{p}{q}$ fraction, for integer p and q

[2 marks]

Question 8

For each of the following series state if the terms are in

- Arithmetic Progression
- Geometric Progression
- Neither Arithmetic nor Geometric Progression

(i) $7 + 3 - 1 - 5 - ...$

(ii) $1 + 8 + 27 + 64 + ...$

(iii) $0.1^3 + 0.1^5 + 0.1^7 + 0.1^9 + ...$

(iv) $3 - 1 + \frac{1}{3} - \frac{1}{9} + ...$

(v) $1 - 1 + 1 - 1 + 1 - 1 + ...$

[5 marks]

Question 9

Determine the value of this series which is in Geometric Progression, and expressed in sigma notation

$$\sum_{1}^{4} 3^n$$

[3 marks]

Question 10

Determine the value of this series which is in Geometric Progression, and expressed in sigma notation

$$\sum_{1}^{5} 3 \times 2^n$$

[3 marks]

Question 11

If 3, x and 9 are the first three terms of a sequence in Geometric Progression, find

(i) the possible exact values of x

[3 marks]

(ii) the possible exact values of the 4^{th} term.

[2 marks]

Question 12

The 7^{th} term of a Geometric Progression is exactly 1.9487171

and the 3^{rd} term is exactly 1.331

(i) What is the common ratio ?

[3 marks]

(ii) Write out the first 6 terms of the Geometric Progression.

[2 marks]

(iii) Express the sum of first 40 terms of this Geometric Progression in sigma notation.

[2 marks]

Question 13

A geometric sequence has first term 4 and third term 1

Find the two possible values of the 6^{th} term.

[4 marks]

Question 14

The first three terms of a geometric sequence are given by

$$8 - x, \qquad 2x, \qquad x^2$$

respectively where $x > 0$

(i) Show that $x^3 - 4x^2 = 0$

[2 marks]

(ii) Find the value of the 20^{th} term.

[3 marks]

(iii) State, with a reason, whether 4096 is a term in the sequence.

[1 mark]