Lesson 2

2.1 The Sum Of A Geometric Progression

It is not too difficult to use a calculator or even simply mentally sum the following series which is in Geometric Progression,

$$4 + 20 + 100 + 500 + 2500 + 12500 + 62500$$

For a Geometric Progressions with more terms the direct method of simply writing out all the terms and then adding then up becomes impractical.

Before tackling this “find the sum” problem as an algebraic entity, it's helpful to look at a numerical method to sum the above Geometric Progression but that can be generalised to derive an algebraic formula.

The key idea is to call the sum of the series, *S*, then multiply *S* by 5.
Where did that 5 come from ?
Well, it is the common ratio of this particular series.

$$\begin{array}{lrl} & 5 \times S = & \quad\;\; 20 + 100 + \dots + 62500 + 312500 \\ \text{SUBTRACT} & S = & 4 + 20 + 100 + \dots + 62500 \\ \hline & 4S = & -4 \qquad\qquad\qquad\qquad\qquad + 312500 \end{array}$$

$$4S = 312496$$

$$S = \frac{312496}{4}$$

$$S = 78124$$

2.2 Formula To Sum A Geometric Progression

The proof of the formula to sum the terms of a Geometric Progression is occasionally asked for in examinations.

What follows is a video from Exam Solutions of the proof.
Over the page is a written out version.

Teaching Video : http://www.NumberWonder.co.uk/v9077/2.mp4

Algebraically, a Geometric Progression is a number sequence of the form;

$$a, \quad ar, \quad ar^2, \quad ar^3, \quad ...., \quad ar^{n-1}$$

where a is the initial term
r is the common ratio
n is number of terms

Notice that the power of the last term is not n but $n - 1$

That is, $G_n = a r^{n-1}$

$$rS = \quad ar + ar^2 + ... + ar^{n-1} + ar^n$$

SUBTRACT $S = a + ar + ar^2 + ... + ar^{n-1}$

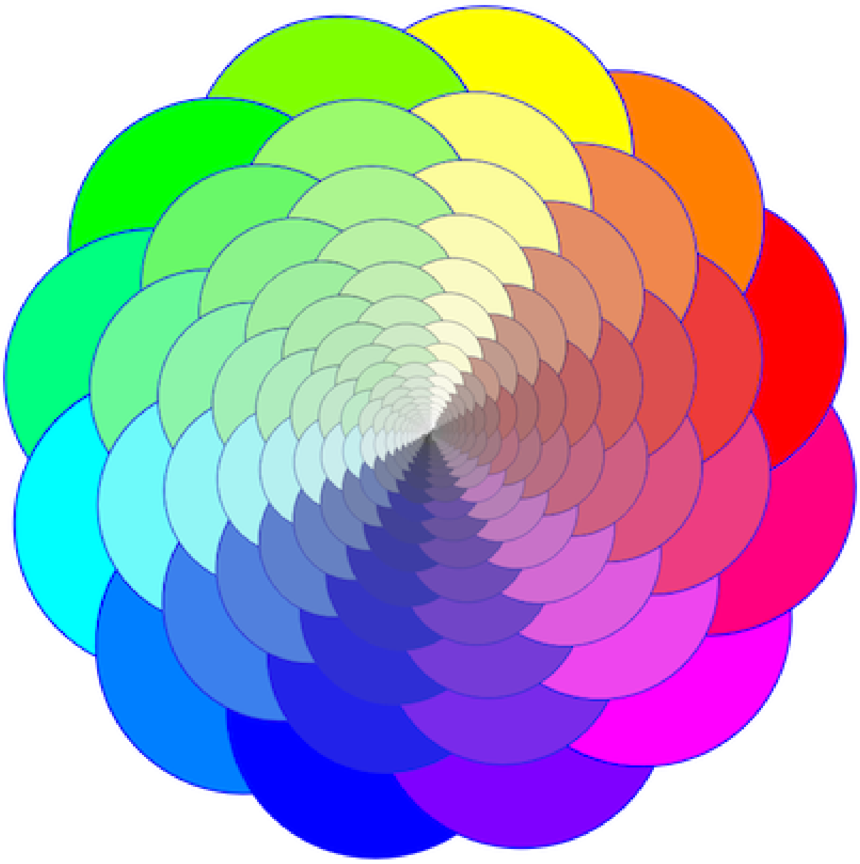
$$rS - S = -a \qquad + ar^n$$

$$(r - 1) S = a(r^n - 1)$$

$$S = \frac{a(r^n - 1)}{(r - 1)} \qquad r \neq 1$$

The above version of the "Sum of a GP" formula is most useful if $r > 1$
If $r < 1$ this alternative, equivalent, version may be easier to work with;

$$S = \frac{a(1 - r^n)}{(1 - r)} \qquad r \neq 1$$



A Geometric Series in Bloom by Nikki Geib

2.3 Exercise

Marks Available: 40

Question 1

Find the sum of the first 10 terms of the following Geometric Progression;

$$3 + 12 + 48 + 192 + \ldots$$

[2 marks]

Question 2

Find the sum of the first 10 terms of the following Geometric Progression;

$$59049 + 19683 + 6561 + 2187 + \ldots$$

[2 marks]

Question 3

(i) Write out the first four terms of the Geometric Progression described by;

$$\sum_{1}^{12} 0.8^{n}$$

[2 marks]

(ii) Determine the exact value of

$$\sum_{1}^{12} 0.8^{n}$$

[2 marks]

Question 4

A geometric series has first four terms,

$$5 - 2 + 0.8 - 0.32 + \ldots$$

Give the exact value of the sum of the first 10 terms

[3 marks]

Question 5

A geometric series has first term *a* and common ratio 2.
A different geometric series has first term *b* and common ratio 3.
Given that the sum of the first 4 terms of both series is the same, show that

$$\frac{a}{b} = \frac{8}{3}$$

[3 marks]

Question 6

A number of plastic bottles being washed up on a beach is estimated to be growing at a rate of 15% a year. In 2020, 400 plastic bottles were washed up.

(i) How many plastic bottles are projected to be washed up in;

(a) 2021

[1 mark]

(b) 2022

[1 mark]

(c) 2023

[1 mark]

(ii) In total how many bottles are projected to be washed up over the ten year period from 2021 - 2029 (inclusive) ?

[3 marks]

Question 7

The first three terms of a geometric series are

$$(k - 6), \quad k, \quad (2k + 5)$$

where k is a positive constant.

(i) Show that $k^2 - 7k - 30 = 0$

[3 marks]

(ii) Hence find the value of k

[2 marks]

(iii) Find the common ratio of this series

[1 mark]

(iv) Find the sum of the first 10 terms of this series, giving your answer to the nearest whole number.

[2 marks]

Question 8

Determine the exact value of

$$\sum_{1}^{12} 7 \times 3^n$$

[3 marks]

Question 9
Find the difference between the sums to ten terms of the arithmetic series and the geometric series whose first two terms are -2 and 4

[4 marks]

Question 10
(i) Show that there are two possible geometric series in each of which the first term is 8, and the sum of the first three terms is 14.

[3 marks]

(ii) Write out the first 6 terms of each series.

[2 marks]