Lesson 3

A-Level Pure Mathematics, Year 2
Geometric Progressions

3.1 The Sum To Infinity

Teaching Video : http://www.NumberWonder.co.uk/v9077/3a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9077/3b.mp4 (Part 2)

<= Part 1 Part 2 =>

For a typical Arithmetic Progression...

$$7 = 7$$
$$7 + 11 = 18$$
$$7 + 11 + 15 = 33$$
$$7 + 11 + 15 + 19 = 52$$

Observation:

For a typical Geometric Progression with either $r > 1$ or $r < 1$...

$$4 = 4$$
$$4 + 20 = 24$$
$$4 + 20 + 100 = 124$$
$$4 + 20 + 100 + 500 = 624$$

Observation:

For a typical Geometric Progression with either $-1 < r < 1$

$$64 = 64$$
$$64 + 32 = 96$$
$$64 + 32 + 16 = 112$$
$$64 + 32 + 16 + 8 = 120$$
$$64 + 32 + 16 + 8 + 4 = 124$$

Question Time !
As more terms are added, will this ever sum to more than

(i) 500 ? **(ii)** 200 ? **(iii)** 130 ?

(iv) 128 ? **(v)** 126 ?

$$64 = 64$$
$$64 + 32 = 96$$
$$64 + 32 + 16 = 112$$
$$64 + 32 + 16 + 8 = 120$$
$$64 + 32 + 16 + 8 + 4 = 124$$
$$64 + 32 + 16 + 8 + 4 + 2 = 126$$
$$64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$$
$$64 + 32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} = 127\frac{1}{2}$$
$$64 + 32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 127\frac{3}{4}$$
$$64 + 32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 127\frac{7}{8}$$

The series is approaching a limit of 128 but never quite gets there.
This series would be described as having a sum to infinity of 128, which is the upper bound of the series, and is the smallest number this series can not sum to.

For a Geometric Progression with $-1 < r < 1$ it makes sense to talk about a sum to infinity because such a series is convergent on a fixed number.

3.2 The Sum To Infinity Formula For A Geometric Progression

$$Sum_{\infty} = \frac{a}{1 - r} \qquad -1 < r < 1$$

3.3 Example

Show that 128 is the sum to infinity of the geometric series,

$$64 + 32 + 16 + \ldots$$

[2 marks]

3.4 Exercise

Marks Available : 46

Question 1

Find S_∞ of the geometric series,

$$12 - 6 + 3 - 1.5 + \ldots$$

[2 marks]

Question 2

A geometric series has first term -5 and sum to infinity -3

Find the common ratio.

[3 marks]

Question 3

For the geometric series with $S_3 = 9$ and $S_\infty = 8$, find the value of the common ratio and also the value of the first term.

[4 marks]

Question 4

C2 Examination question from June 2009, Q5.

The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$

[2 marks]

(b) Find the first term of the sequence

[2 marks]

(c) Find the sum of the first 15 terms of the sequence

[3 marks]

(d) Find the sum to infinity of the sequence

[2 marks]

Question 5

C2 Examination question from January 2007, Q10

A geometric series is

$$a + ar + ar^2 + \ldots$$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

[4 marks]

(b) Find $\sum_{k=1}^{10} 100\left(2^k\right)$

[3 marks]

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots$$

[3 marks]

(d) State the condition for an infinite geometric series with common ratio r to be convergent

[1 mark]

Question 6

C2 Examination question from January 2009, Q9

The first three terms of a geometric series are

$$(k + 4), \quad k, \quad (2k - 15)$$

where k is a positive constant.

(i) Show that $k^2 - 7k - 60 = 0$

[4 marks]

(ii) Hence show that $k = 12$

[2 marks]

(iii) Find the common ratio of this series

[1 mark]

(iv) Find the sum to infinity of this series.

[2 marks]

Question 7

C2 Examination question from January 2005, Q6

The second and fourth terms of a geometric series are 7.2 and 5.832 respectively.

The common ratio of the series is positive.

For this series, find

(a) the common ratio

[2 marks]

(b) the first term

[2 marks]

(c) the sum of the first 50 terms, giving your answer to 3 decimal places

[2 marks]

(d) the difference between the sum to infinity and the sum of the first 50 terms, giving your answer to 3 decimal places

[2 marks]