

Lesson 3

A-Level Pure Mathematics : Year 1 Algebra of Surds and Indices II

3.1 Equations Involving Surds & Indices

Example #1 : Index Matching

Given that

$$81\sqrt{3} = 3^a$$

find the value of a .

[2 marks]

Example #2 : Index Matching

Given that

$$(27x^{12})^{\frac{5}{3}} = 3^a x^b$$

find the value of a and the value of b .

[2 marks]

Example #3 : Surd Coefficient Matching

Given that,

$$(3 + \sqrt{c})(2\sqrt{c} - 3) = 1 + k\sqrt{c}$$

where c and k are prime numbers, find the value of c and the value of k

[3 marks]

Example #4 : Double Square Root Surds

Given that,

$$\sqrt{10 + 2\sqrt{21}} = \sqrt{a} + \sqrt{b} \quad \text{with } a < b$$

find the value of a and the value of b , both of which are integers.

[3 marks]

3.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 55

Question 1

Without using a calculator, write down the value of each of the following.

Answers should be written as exact simplified rational numbers without indices.

(i) $25^{\frac{1}{2}}$ (ii) $27^{\frac{1}{3}}$ (iii) 3^{-2} (iv) $4^{\frac{3}{2}}$ (v) $9^{-\frac{1}{2}}$

(vi) $100^{-\frac{3}{2}}$ (vii) $81^{\frac{1}{4}}$ (viii) $81^{\frac{3}{4}}$ (ix) $81^{-\frac{1}{2}}$ (x) 81^0

[5 marks]

Question 2

Given that a , b and c are distinct prime numbers and that

$$y = 2^5 \times 3^4 \times 5^3$$

determine the value of \sqrt{y}

Write your answer in the form $a\sqrt{b}$ where a and b are integers and b is \square free.

[2 marks]

Question 3

Without using a calculator, write down the value of each of the following.
Answers should be written as simplified exact numbers without indices and with denominators that are rational.

$$(i) \quad \left(\frac{4}{7}\right)^2 \qquad (ii) \quad \left(\frac{3}{2}\right)^{-2} \qquad (iii) \quad \left(\frac{22}{77}\right)^0$$

$$(iv) \quad \left(\frac{3}{8}\right)^{-1} \qquad (v) \quad \left(\frac{50}{32}\right)^{\frac{1}{2}}$$

[5 marks]

Question 4

Without using a calculator, write down the value of each of the following.
Answers should be written as simplified exact numbers without indices and with denominators that are rational.

$$(i) \quad \left(-\frac{16}{54}\right)^{-\frac{1}{3}} \qquad (ii) \quad \left(\frac{9}{6\sqrt{2}}\right)^3$$

$$(iii) \quad \left(\frac{25}{8}\right)^{\frac{1}{2}} \qquad (iv) \quad \left(\frac{7 + \sqrt{5}}{8}\right)^{-1}$$

[4 marks]

Question 5

Given that $8\sqrt{2} = 2^a$ find the value of a

[2 marks]

Question 6

Given that,

$$\sqrt{3 + 2\sqrt{2}} = \sqrt{a} + \sqrt{b} \quad \text{with } a > b$$

find the value of a and the value of b , showing clear reasoning to justify your answer.

[3 marks]

Question 7

Given that,

$$(7 - \sqrt{c}) (4 + 2\sqrt{c}) = 6 + k\sqrt{c}$$

where c and k are integers and c is square free, find the value of c and the value of k

[3 marks]

Question 8

Showing all steps in your reasoning, work out the exact value of n , given that

$$\frac{1}{\sqrt[3]{9^4}} = 3^n$$

[2 marks]

Question 9

Given that

$$(a + \sqrt{5})(3 + 2\sqrt{5}) = 31 + b\sqrt{5}$$

find the value of a and the value of b both of which are integers.

[3 marks]

Question 10

Given that,

$$x = \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$$

prove that x has the value 2, exactly.

[4 marks]

Question 11

Given that for some prime, p ,

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m

[2 marks]

Question 12

Two composite numbers, g and h have prime number decompositions

$$g = a^3 \times b \times c^2 \qquad h = a \times b \times c^3$$

where a , b and c are distinct prime numbers.

- (a) Express gh as a product of powers of its prime factors
Simplify your answer

[2 marks]

- (b) Find the value of x , the value of y and the value of z given that,

$$\frac{g}{h} = a^x \times b^y \times c^z$$

[3 marks]

Question 13

Given that,

$$\sqrt{8 - 4\sqrt{3}} = \sqrt{a} - \sqrt{b} \qquad \text{with } a > b$$

find the value of a and the value of b , both of which are integers

[3 marks]

Question 14

GCSE Examination Question, January 2017, Paper 3H(R), Q18

Given that p is a prime number, rationalise the denominator of

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}}$$

Simplify your answer

[3 marks]

Question 15

A-Level Examination Question, June 2019, Paper 2, Q1

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x .

[3 marks]

Question 16

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3 \times 5^4$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is \square free.

[2 marks]

Question 17

Carefully showing your working, rationalise the denominator of,

$$\frac{1}{1 + \sqrt{2} + \sqrt{3}}$$

[4 marks]

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