

## Lesson 11

### A-Level Pure Mathematics : Year 2 Differentiation IV

#### 11.1 Revision

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 50

#### Question 1

Differentiate each of the following, simplifying answers as appropriate;

(i)  $y = \frac{2x^5}{15}$

[ 2 marks ]

(ii)  $y = \ln(2x^3 + 7)$

[ 2 marks ]

(iii)  $y = \frac{5}{4x^2 - 3}$

[ 2 marks ]

(iv)  $y = e^{\sqrt{x}}$

[ 2 marks ]

**Question 2**

The product rule states that  $(uv)' = uv' + u'v$

Use the rule to differentiate  $y = 7x^2 \cos x$

[ 3 marks ]

**Question 3**

The quotient rule states that  $\left(\frac{u}{v}\right)' = \frac{vu' - v'u}{v^2}$

Use the rule to differentiate the following, simplifying your answer;

$$y = \frac{\ln(4x)}{x^2}$$

[ 3 marks ]

**Question 4**

(i) Use derivatives of  $\sin x$  and  $\cos x$  to prove the derivative of  $\tan x$  is  $\sec^2 x$

[ 4 marks ]

(ii) Hence, or otherwise, use the chain rule to differentiate;

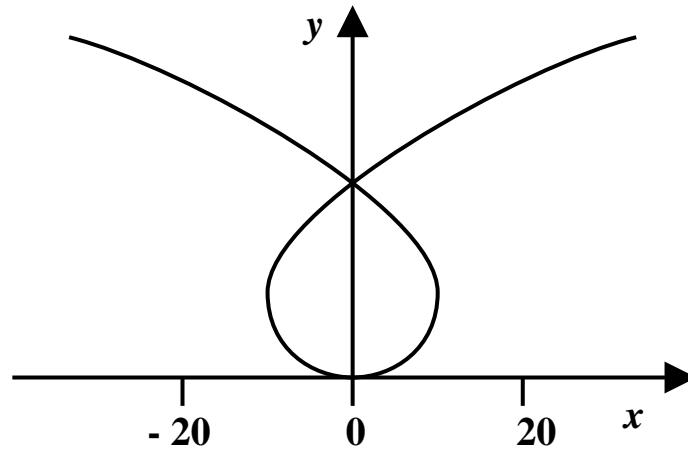
$$y = \tan^2 x$$

[ 2 marks ]

**Question 5**

The graph is of the parametric equations;

$$x = 12t - t^3 \quad \text{and} \quad y = 3t^2$$



(i) Find, in terms of  $t$ ,

(a)  $\frac{dx}{dt}$

(b)  $\frac{dy}{dt}$

(c)  $\frac{dy}{dx}$

[ 2, 2, 2 marks ]

(ii) Write down the coordinates of the point on the curve that corresponds to the parameter  $t$  having the value 1

[ 1 mark ]

(iii) What is the gradient of the curve at your part (ii) point ?

[ 1 mark ]

(iv) By making use of your part (ii) and (iii) answers, determine the equation of the tangent to the curve from the point at which  $t = 1$

[ 2 marks ]

**Question 6**

A curve has equation;

$$x^2 + 6xy - y^2 = 90$$

Find an expression for the gradient by means of implicit differentiation.

Write your answer in the form  $\frac{dy}{dx} = f(x, y)$

[ 6 marks ]

**Question 7**

The parametric equations of a curve are;

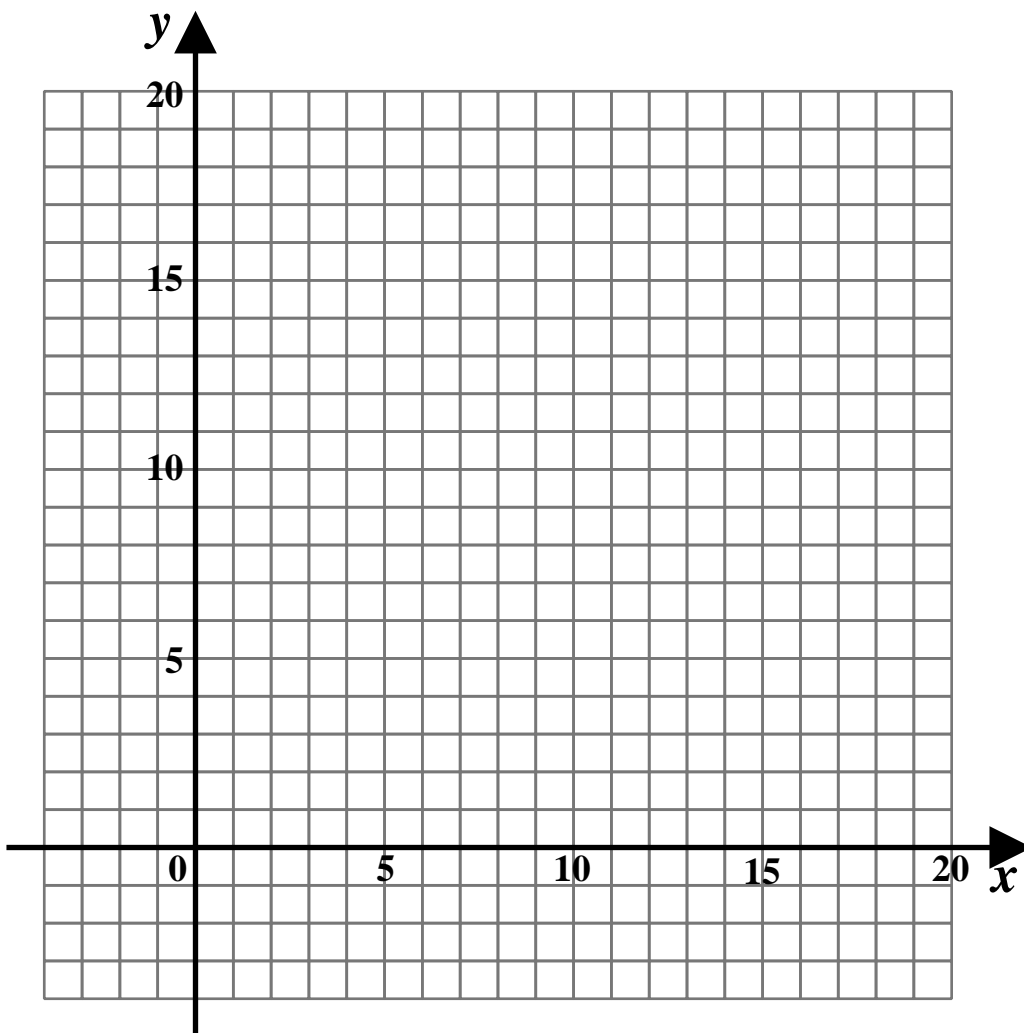
$$x = t^2 + t, \quad y = t^2 - t$$

- (i) Complete the following table by way of working out some points on the graph of this curve.

$t$	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
$x$											
$y$											

[ 3 marks ]

- (ii) On the graph paper provided below plot the curve



[ 3 marks ]

( iii ) Find, in terms of  $t$ , an expression for the derivative of this curve.

[ 4 marks ]

( iv ) Find, in terms of  $x$  and  $y$  an expression for the derivative of this curve.

[ 4 marks ]

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