Lesson 13

A-Level Pure Mathematics : Year 2

Differentiation IV

13.1 Differentiation : Later Date Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Without looking up the formulae booklet, differentiate each of the following with respect to x. If you can remember a formulae book formula you can use it !

(i) $y = e^{5x}$

(ii) $y = 4 \sin 6x$

(iii) $y = \cos^2 3x$

(iv) $y = \tan x$

(v) $y = \dfrac{1}{\cos x}$

(vi) $y = \ln\left(5x^3\right)$

(vii) $y = (\ln(5x))^3$

[14 marks]

Question 2

Use the product rule to find the derivative with respect to x of $y = 3x\,e^{5x}$ giving the answer in the form $\frac{dy}{dx} = A\,e^{5x}(Bx + C)$ where A, B and C are integers to be found.

[3 marks]

Question 3

Use the quotient rule to find the derivative with respect to x of $y = \frac{sin\,3x}{2\,x^2}$ and simplify your answer.

[4 marks]

Question 4

Use the chain rule to find the derivative with respect to x of $y = \left(1 + cos^2 3x\right)^5$

Write your answer in the form $\frac{dy}{dx} = A\left(1 + cos^2 3x\right)^B sin\,Cx$ where A, B, and C are integers to be found.

[3 marks]

Question 5

A curve is described parametrically by the equations

$$x = t - \cos^2 t \qquad\qquad y = \sin^2 t$$

(i) Show that, $\dfrac{dy}{dx} = \dfrac{\sin 2t}{1 + \sin 2t}$

[4 marks]

(ii) Show that when $t = \dfrac{\pi}{6}$, $\dfrac{dy}{dx} = m\sqrt{3} + n$

for some integer values of m and n that you should determine.

[4 marks]

Question 6

Find the tangent to the curve $y = \sin x$ when $x = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$ and give the exact value for c.

[4 marks]

Question 7

Differentiate implicitly to find $\frac{dy}{dx}$ for the curve $4x^3 + 5y^4 + 7xy = 0$

[4 marks]