

2.1 Parametric to Cartesian Manipulations

It may be possible to convert a pair of parametric equations into Cartesian form.

Example #1

Consider the following parametric equations which describe an ellipse[†]

$$x = 4 \sin \theta$$

$$y = 6 \cos \theta$$

In this case it is possible to eliminate the parameter θ , and obtain a single equation containing only numbers and the variables x and y .

Teaching Video : <http://www.NumberWonder.co.uk/v9081/2a.mp4>

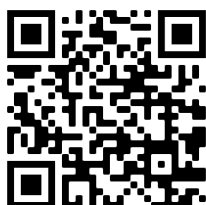
**[4 marks]****Example #2**

Express in the form $y^2 = f(x)$ the following parametric equations[‡]

$$x = 4t^2$$

$$y = 16t(t^2 - 1)$$

Teaching Video : <http://www.NumberWonder.co.uk/v9081/2b.mp4>

**[4 marks]**

[†] These parametric equations were graphed in Lesson 1, Exercise 1.2, Question 3

[‡] These parametric equations were graphed in Lesson 1, Exercise 1.2, Question 2

2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Find an equation of the form $ax^2 + by^2 = c$, where a , b and c are integer constants to be found, for the following pair of parametric equations;

$$x = 15 \sin \theta$$

$$y = 20 \cos \theta$$

[4 marks]

Question 2

Find the Cartesian equations of each of these curves in the form $y = f(x)$

(i) $x = 8t$

$$y = \frac{8}{t}$$

(ii) $x = \frac{20}{t}$

$$y = t^2$$

[3, 3 marks]

(iii) $x = 14t$

$$y = 7t^2$$

(iv) $x = t - 2$

$$y = t^2 + 3$$

[3, 3 marks]

Question 3

Use trigonometric identities to find the Cartesian equations of each of these curves,

(i) $x = 2 \cos \theta$

$$y = 3 \sin \theta$$

(ii) $x = 3 \sec \theta$

$$y = 5 \tan \theta$$

[3, 3 marks]

Question 4

Find the Cartesian equations of each of these curves in the form $y = f(x)$

(i) $x = 2 + 3t$

$$y = \frac{1}{t}$$

(ii) $x = 3 + 2t$

$$y = 4t^2 - 9$$

[3, 3 marks]

Question 5

Find the Cartesian equations of the curve with parametric equations ;

$$x = 2 \cos \theta$$

$$y = 5 \sin 2\theta$$

Give your answer in the form $y^2 = f(x)$

Hint : Make use of the trigonometric identity $\sin 2\theta = 2 \sin \theta \cos \theta$

[6 marks]

Question 6

A curve C has parametric equations

$$x = 2 \sin \theta$$

$$y = 1 - \cos 2\theta$$

Find a Cartesian equation for C in the form $y = f(x)$

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk