Lesson 2

A-Level Pure Mathematics : Year 2
Differentiation IV

2.1 Parametric to Cartesian Manipulations

It may be possible to convert a pair of parametric equations into Cartesian form.

Example #1

Consider the following parametric equations which describe an ellipse†

$$x = 4 \sin \theta$$

$$y = 6 \cos \theta$$

In this case it is possible to eliminate the parameter θ, and obtain a single equation containing only numbers and the variables x and y.

Teaching Video : http://www.NumberWonder.co.uk/v9081/2a.mp4

[4 marks]

Example #2

Express in the form $y^2 = f(x)$ the following parametric equations‡

$$x = 4t^2$$

$$y = 16t(t^2 - 1)$$

Teaching Video : http://www.NumberWonder.co.uk/v9081/2b.mp4

[4 marks]

† These parametric equations were graphed in Lesson 1, Exercise 1.2, Question 3
‡ These parametric equations were graphed in Lesson 1, Exercise 1.2, Question 2

2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Find an equation of the form $a x^2 + b y^2 = c$, where a, b and c are integer constants to be found, for the following pair of parametric equations;

$$x = 15 \sin \theta$$

$$y = 20 \cos \theta$$

[4 marks]

Question 2

Find the Cartesian equations of each of these curves in the form $y = f(x)$

(i) $x = 8t$

$y = \frac{8}{t}$

(ii) $x = \frac{20}{t}$

$y = t^2$

[3, 3 marks]

(iii) $x = 14t$

$y = 7t^2$

(iv) $x = t - 2$

$y = t^2 + 3$

[3, 3 marks]

Question 3

Use trigonometric identities to find the Cartesian equations of each of these curves,

(i) $x = 2\cos\theta$, $y = 3\sin\theta$ **(ii)** $x = 3\sec\theta$, $y = 5\tan\theta$

[3, 3 marks]

Question 4

Find the Cartesian equations of each of these curves in the form $y = f(x)$

(i) $x = 2 + 3t$, $y = \frac{1}{t}$ **(ii)** $x = 3 + 2t$, $y = 4t^2 - 9$

[3, 3 marks]

Question 5

Find the Cartesian equations of the curve with parametric equations ;

$$x = 2\cos\theta$$

$$y = 5\sin 2\theta$$

Give your answer in the form $y^2 = f(x)$

Hint : Make use of the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$

[6 marks]

Question 6

A curve *C* has parametric equations

$$x = 2\sin\theta$$

$$y = 1 - \cos 2\theta$$

Find a Cartesian equation for *C* in the form $y = f(x)$

[6 marks]