3.1 Differentiation Of Parametric Equations

To differentiate parametric equations two ideas are needed,

The Two Parametric Differentiation Key Facts

- $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$ The Chain Rule
- $\dfrac{d\theta}{dx} = \dfrac{1}{\left(\frac{dx}{d\theta}\right)}$ The Inversion Rule

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A proper mathematically rigorous proof of either of these is difficult and tricky but the two results are easily remembered because it is as if the entities involved are behaving like ordinary fractions. For A-Level you should think of them this way, (as ordinary fractions) but keep in mind that they are “not really” and that there is an unresolved issue here for tackling on a University Maths course !

Example

Given the parametric equations $x = sin\,\theta$

$$y = cos\,\theta$$

(i) Sketch the part circle traced out as θ increases from 0 to π radians.

(ii) Find $\dfrac{dy}{dx}$ in terms of θ

(iii) State the value of the gradient when $\theta = \dfrac{3\pi}{4}$

Teaching Video : http://www.NumberWonder.co.uk/v9081/3.mp4

[2, 3, 1 marks]

3.2 Table of standard derivatives

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes

When trigonometry and calculus mix, θ must be in **RADIANS**

3.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 50

Question 1

Given that $x = 5t^4$

$y = 3t^5$

Show that

$$\frac{dy}{dx} = \frac{3}{4}t$$

[3 marks]

Question 2

Given that $x = t^2 + t$

$y = 2t - t^2$

Find $\frac{dy}{dx}$ in terms of t

[3 marks]

Question 3

Given that $x = (4t + 2)^3$

$y = (3t + 4)^2$

Find $\frac{dy}{dx}$ in terms of t

[3 marks]

Question 4

Given that $x = 2\cot\theta$

$y = 3\tan\theta$

Show that

$$\frac{dy}{dx} = -\frac{3}{2}\tan^2\theta$$

[3 marks]

Question 5

Find $\frac{dy}{dx}$ in terms of θ for the parametric equations $x = 3\sin\theta$

and $y = 5\cos\theta$

[3 marks]

Question 6

For the parametric equations $x = tan\,\theta$

and $y = sec\,\theta$

(i) Find $\frac{dy}{dx}$ in terms of θ

[3 marks]

(ii) What is the value of the gradient when θ is $\frac{\pi}{3}$ radians ?

[1 mark]

Question 7

The rule for the differentiation of products tells us that;

If $f = u\,v$ then $f\,' = u\,v' + u'\,v$

Consider the parametric equations $x = -\,cos\,\theta$

and $y = \theta\,sin\,\theta$

Show that $\frac{dy}{dx} = \theta\,cot\,\theta + 1$

[4 marks]

Question 8

For the parametric equations $x = \cot\theta$

and $y = \csc\theta$

(i) Find $\frac{dy}{dx}$ in terms of θ

[3 marks]

(ii) What is the value of the gradient when θ is $\frac{\pi}{3}$ radians ?

[1 mark]

Question 9

For the parametric equations $x = -\sin\theta$

and $y = \theta\cos\theta$

Find $\frac{dy}{dx}$ in terms of θ

[4 marks]

Question 10

Consider the parametric equations $x = 5t^2$

and $y = 4t^3$

(i) Determine the value of x and the value of y when $t = 2$.
This is a point on the curve.

[1 mark]

(ii) Find $\dfrac{dy}{dx}$ in terms of t

[3 marks]

(iii) Use your part (ii) answer to show that when $t = 2$, $\dfrac{dy}{dx} = \dfrac{12}{5}$

[2 marks]

(iv) Use your part (i) point, and your part (iii) gradient, to find the equation of the tangent to the curve when $t = 2$.
Give the answer in the form $ax + by + c = 0$
where a, b and c are INTEGERS to be found.

[3 marks]

Question 11

A curve has the parametric equations $x = \cos t$

and $y = \sin 2t$

(i) Determine the value of x and the value of y when $t = \dfrac{\pi}{6}$

This is a point on the curve.

[1 mark]

(ii) Find $\dfrac{dy}{dx}$ in terms of t

[4 marks]

(iii) Use your part (ii) answer to show that when $t = \dfrac{\pi}{6}$, $\dfrac{dy}{dx} = -2$

[2 marks]

(iv) Use your part (i) point, and your part (iii) gradient, to find the equation of the tangent to the curve when $t = \dfrac{\pi}{6}$

Give an exact answer in the form $y = mx + c$ where m and c are constants.

[3 marks]