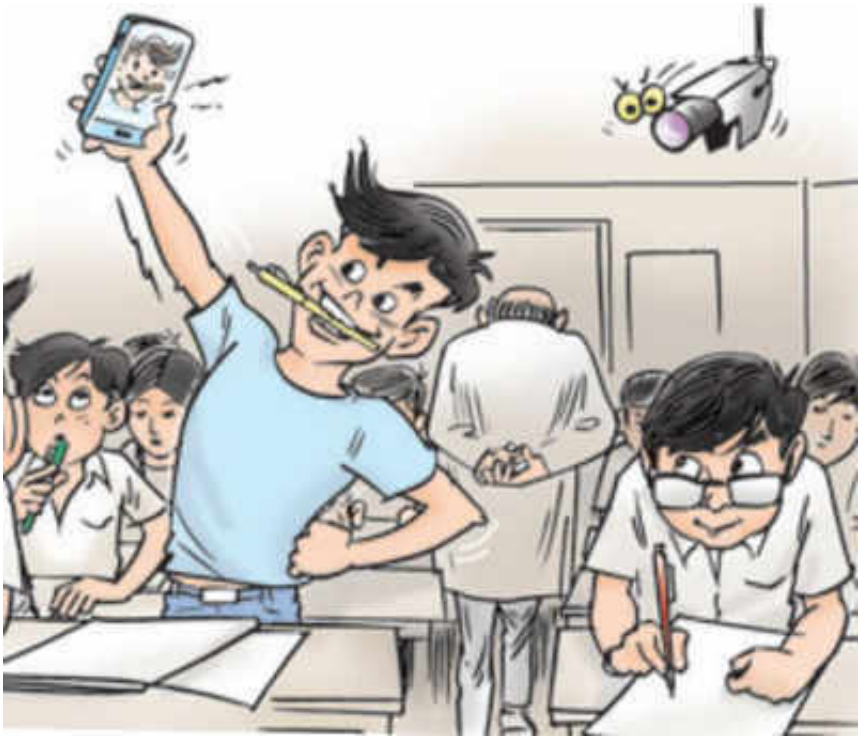
Lesson 5

A-Level Pure Mathematics : Year 2

Differentiation IV

5.1 From The Examination Hall



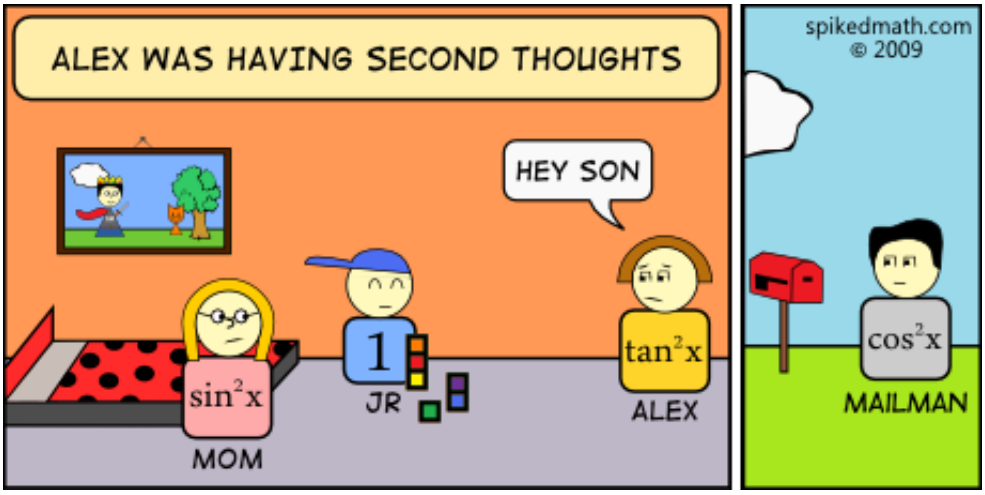
Exam Regulation #284 : No taking of selfies in the Examination Hall

It is highly likely that in your examination there will be a question involving parametric equations and so this lesson is a further practice session using old examination questions.

In the examination you may look up the derivatives of many functions in the examination formulae book but don't forget that many of the most useful trig identities are not given so you either need to remember how to derive them or commit them to memory.

5.2 The NOT Provided Trigonometric Identities

$\cos^2\theta + \sin^2\theta = 1$		$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$1 + \tan^2\theta = \sec^2\theta$		$\sin 2\theta = 2\sin\theta\cos\theta$
$\cot^2\theta + 1 = \csc^2\theta$		$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$



5.3 Table of Standard Derivatives

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes

When trigonometry and calculus mix, θ must be in **RADIANS**

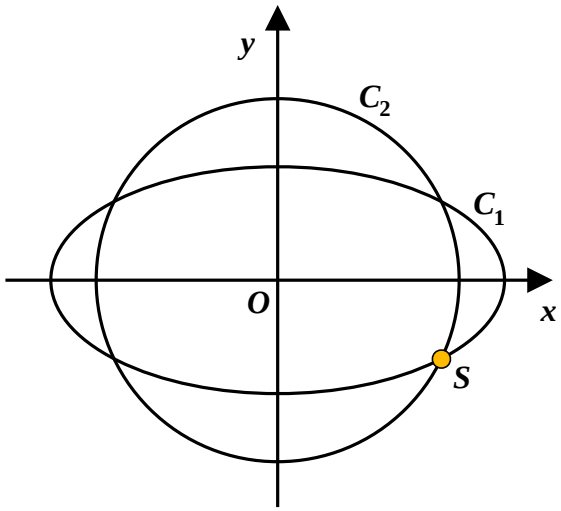
5.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Question 1

A-Level Examination Question from June 2019, Paper 2, Q4 (Edexcel)



The curve C_1 with parametric equations

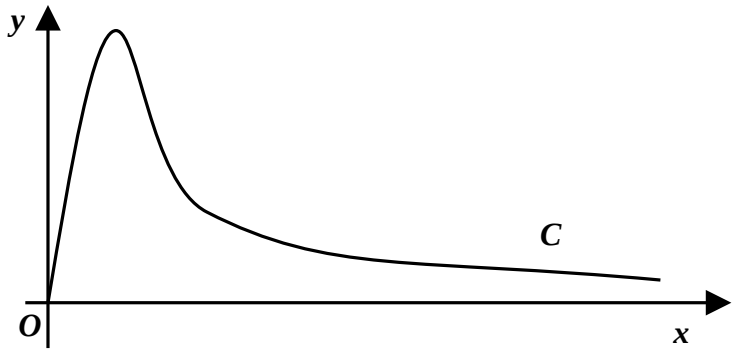
$$x = 10\cos t, \qquad y = 4\sqrt{2}\sin t, \qquad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation $x^2 + y^2 = 66$ at four distinct points as shown. Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S

[6 marks]

Question 2

A-Level Examination Question from June 2016, Paper C4, Q5 (Edexcel)



The sketch is of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd.

[4 marks]

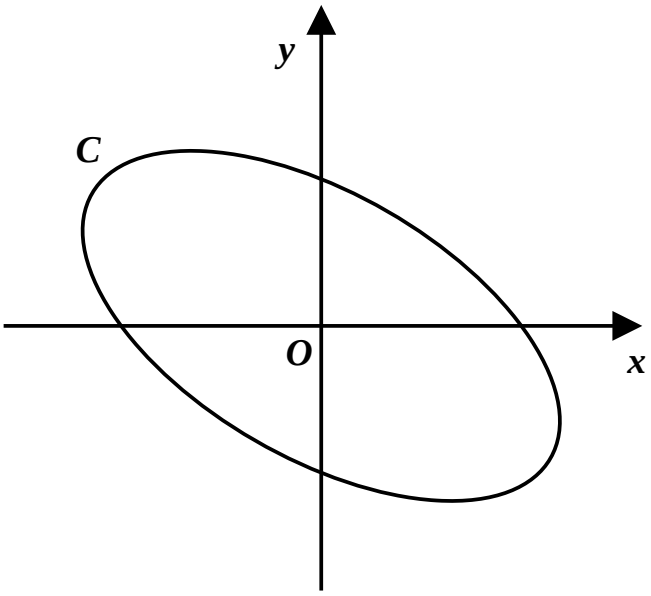
The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q

[2 marks]

Question 3

A-Level Specimen Paper 1 for the June 2021 (cancelled) Examination, Q14 (Edexcel)



The sketch is of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \qquad y = 2\sin t, \qquad 0 < t \leqslant 2\pi$$

Show that a Cartesian equation of C can be written in the form

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

[5 marks]

Question 4

The curve C has parametric equations

$$x = \sec t, \qquad y = \tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Prove that $\frac{dy}{dx} = \csc t$

[4 marks]

(b) Find the equation in the form $y = mx + c$ of the tangent to C at the point where $t = \frac{\pi}{4}$

[4 marks]

Question 5

A-Level Examination Question from June 2010, Paper C4, Q4 (Edexcel)

A curve C has parametric equations

$$x = \sin^2 t, \qquad y = 2 \tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t

[4 marks]

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P

[6 marks]

Question 6

A-Level Examination Question from June 2018, Paper 1, Q14 (Edexcel)

A curve C has parametric Equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points of C satisfy $y = 6 - (x - 3)^2$

[2 marks]

(b) **(i)** Sketch the curve C

(ii) Explain briefly why C does not include all points of

$$y = 6 - (x - 3)^2, \qquad x \in \mathbb{R}$$

[3 marks in total for (b)]

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

[5 marks]