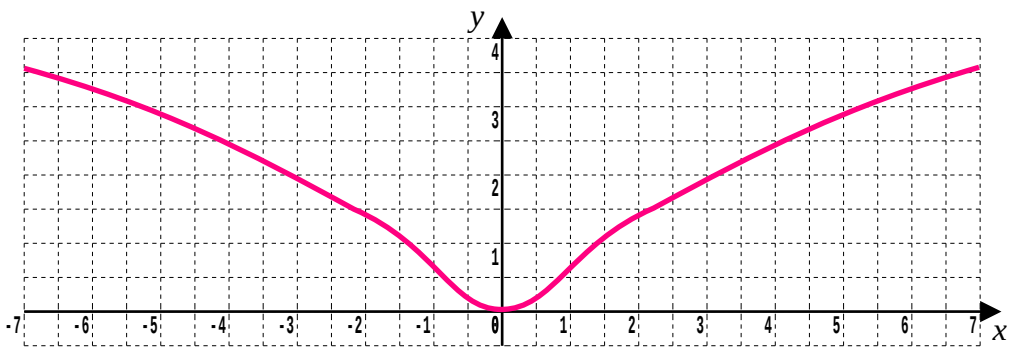
Lesson 7

A-Level Pure Mathematics : Year 2

Differentiation IV

7.1 Implicit Differentiation

The graph is of the equation $y^3 = x^2 - y$ which, although not particularly complicated, is not writable in the form $y = f(x)$. As a consequence its not a routine task to find an expression for the gradient of this curve using the techniques studied thus far.



Implicit differentiation is a method of sweeping through an equation as it stands with a view to obtaining an expression for $\frac{dy}{dx}$ in terms of x and y.

7.2 Example

Obtain an equation of the form $\frac{dy}{dx} = f(x, y)$ for the curve $y^3 = x^2 - y$

Teaching Video: http://www.NumberWonder.co.uk/v9081/7.mp4

[4 marks]

7.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

(i) Obtain an equation of the form $\frac{dy}{dx} = f(x, y)$ for the curve,

$$y^3 + 3y - x^3 - x = 130$$

[5 marks]

(ii) Verify that the point $P(2, 5)$ is on the curve.

[2 marks]

(iii) Show that the gradient at the point $P(2, 5)$ is $\frac{1}{6}$

[2 marks]

(iv) Determine the equation of the tangent to the curve at the point P Give your answer in the form $ax + by + c = 0$ where a, b and c are integer constants.

[3 marks]

Question 2

Use implicit differentiation to show that $y^4 = \sin(x) + \sin(y)$

has the derivative $\frac{dy}{dx} = \frac{\cos(x)}{4y^3 - \cos(y)}$

[4 marks]

Question 3

A-Level Examination Question from January 2009, Paper C4, Q1 (Edexcel)

A curve C has the equation $y^2 - 3y = x^3 + 8$

(a) Find $\frac{dy}{dx}$ in terms of x and y

[4 marks]

(b) Hence find the gradient of C at the point where $y = 3$

[3 marks]

Question 4

A-Level Examination Question from June 2006, Paper C4, Q1 (Edexcel)

A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0$$

Find an equation of the normal to C at the point $(0, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[7 marks]