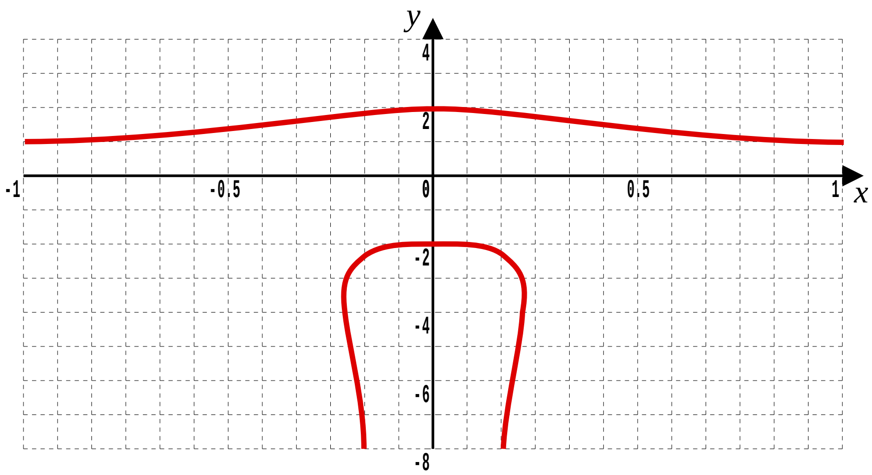
Lesson 8

A-Level Pure Mathematics : Year 2
Differentiation IV

8.1 Product Rule Implicitly

The graph is of the equation

$$4x^2y^3 = 4 + x^2 - y^2$$



The equation is a tangle of x and y, each varying and depending on the other.
To find the gradient of this curve will require implicit differentiation but the term on the left hand side is a product.
Full marks for thinking “No problem, I can use the product rule” !

The Product Rule

$$\text{If } f = uv \quad \text{then} \quad f' = uv' + u'v$$

8.2 Example

Obtain an equation of the form $\dfrac{dy}{dx} = f(x, y)$ for the curve $4x^2y^3 = 4 + x^2 - y^2$

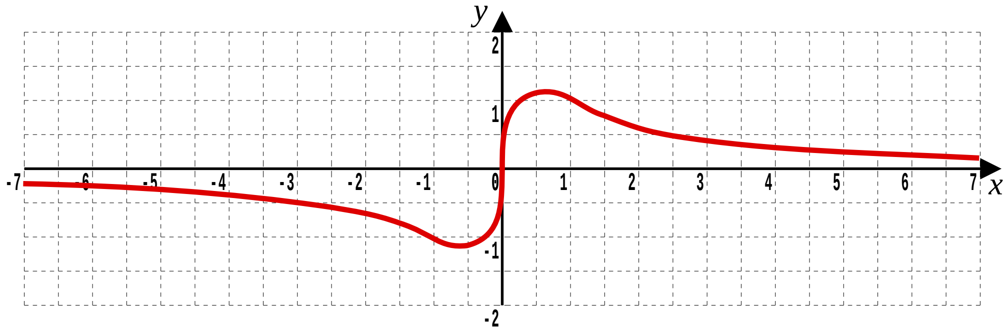
Teaching Video: http://www.NumberWonder.co.uk/v9081/8.mp4

[4 marks]

8.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 70

Question 1



(i) Obtain an equation of the form $\frac{dy}{dx} = f(x, y)$ for the curve,

$$3x^2 y = 4x - y^3$$

[6 marks]

(ii) Verify that the point $Q(1, \ 1)$ is on the curve.

[2 marks]

(iii) Show that the gradient at the point $Q(1, \ 1)$ is $-\frac{1}{3}$

[2 marks]

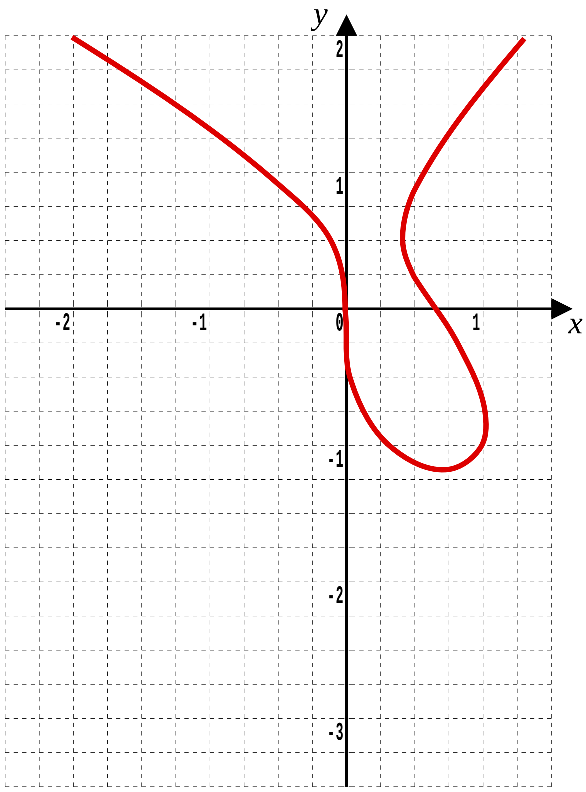
(iv) Determine the equation of the tangent to the curve at the point P
Give your answer in the form $ax + by + c = 0$ where a, b and c are integer constants.

[2 marks]

Question 2

The graph is of the equation

$$2xy = y^3 + 2x - 3x^2$$



(i) Obtain an equation of the form $\dfrac{dy}{dx} = f(x, y)$ for the curve,

$$2xy = y^3 + 2x - 3x^2$$

[6 marks]

(ii) From scrutiny of the graph it looks as if $R\left(-2,\ 2\right)$ is a point with integer coordinates that is on the graph.
Verify that $R\left(-2,\ 2\right)$ is indeed on the curve.

[2 marks]

(iii) From looking at the graph, find another point with integer coordinates other than $\left(0,\ 0\right)$ that is on the curve.
Use mathematics to verify that your point is indeed on the curve.

[3 marks]

(iv) Find the gradient at your part (iii) point.

[2 marks]

(v) Determine the equation of the tangent to the curve at your part (iii) point.

[2 marks]

(vi) Draw your tangent onto the graph, paying particular attention to where it crosses the *y*-axis.

[3 marks]

Question 3

A-Level Examination Question from January 2012, Paper C4, Q1 (Edexcel)

The curve C has the equation

$$2x + 3y^2 + 3x^2 y = 4x^2$$

The point P on the curve has coordinates $(-1,\ 1)$

(a) Find the gradient of the curve at P

[5 marks]

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

Question 4

A-Level Examination question from January 2006, Paper C4, Q1 (Edexcel)

The curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers

[7 marks]

Question 5

A-Level Examination Question from June 2005, Paper C4, Q2 (Edexcel)

A curve *C* has equation

$$x^2 + 2xy - 3y^2 + 16 = 0$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$

[7 marks]

Question 6

A-Level Examination Question from January 2008, Paper C4, Q5 (Edexcel)

A curve C is described by the equation

$$x^3 - 4y^2 = 12xy$$

(a) Find the coordinates of the two points on the curve where $x = -8$

[3 marks]

(b) Find the gradient of the curve at each of these points

[6 marks]

Question 7

A-Level Examination Question from June 2008, Paper C4, Q4 (Edexcel)

A curve has equation

$$3x^2 - y^2 + xy = 4$$

The points P and Q lie on the curve.

The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q

[6 marks]

(b) Find the coordinates of P and Q

[3 marks]

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